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THERMAL CONDUCTIVITY OF COPPER.

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Part II.

CONDUCTIVITY AT LOW TEMPERATURES.

By R. W. Quick and B. S. Lanphear.

In Part I. appeared an account of an investigation on conductivity of copper between the range of temperature 70° to 170°. As was there stated, many investigations have been made relative to the subject at ordinary and high temperatures, but, to the knowledge of the writers, no experiments had hitherto been made with the object of determining the absolute value of the thermal conductivity K and its dependence on temperature at temperatures below the freezing point. This fact induced the writers to extend their observations on the copper bar already described, to temperatures as low as −60° C.

I.

The Measurement of Temperature.

In dealing with low temperatures, the method employed for measuring the temperature of the bar has to be considerably modi-

fied; for the Wheatstone slide wire bridge cannot, in this case, be calibrated *directly* for temperatures of the bar by means of a mercury thermometer, as was done in the first part of this investigation. Therefore, in addition to the requirement of a means of detection of small changes of resistance, the measurement of the low temperatures of the bar involves: (1) a knowledge of the temperature coefficient of resistance of the wire on the collar through the range 0° to −60°, and (2) a knowledge of the temperature difference between the wire on the collar and the bar beneath it, if such difference exists.

(*a*) *Determination of the temperature coefficient of the wire.*

Now if the curve of resistances and temperatures of the wire be a straight line, the determination of the coefficient α would involve only a measurement of resistance of the wire corresponding to two known temperatures. Dewar and Fleming,[1] who investigated resistances of various metals from high temperatures to nearly −200° C., found that the curve of resistance and temperature of electrolytic copper wire was almost exactly a straight line, with a coefficient 0.00410. In consequence of this fact, together with the fact that experiments on the wire, conducted by the writers, at temperatures above the freezing point, seemed to give a fairly straight line, it was decided to obtain the resistance of a portion of the wire at the temperatures of melting ice and melting mercury (the latter temperature being well established), to calculate α from these observations, and by observing the temperature corresponding to a certain resistance of the wire on the collar, draw a straight line at proper pitch through the point thus located, and to use this curve in reading temperatures corresponding to any resistance. The slide wire bridge was of course easily calibrated for resistances by aid of a standard Wheatstone plug bridge.

To obtain the temperature of the wire at the freezing point of mercury, the following arrangement was used, which proved in every way successful: Referring to Fig. 6, t is a tube about 1 cm. long and 7 mm. in diameter, open at one end, and constructed from very thin sheet copper. On the outside of this cylindrical

[1] London Electrician, v. 29.

tube, and separated from it only by tissue paper to secure good insulation, a portion of the fine insulated copper wire was wound in a single layer, having its terminals joined to very heavy wires which passed through holes in the cork of a glass tube t' about 5 cm. long and 2 cm. in diameter, into which the small copper tube t was placed, as shown in the figure. The glass tube was then enclosed in a glass bottle T, with an air space of about 1 cm. between the two. In the copper tube was placed a small globule of pure mercury.

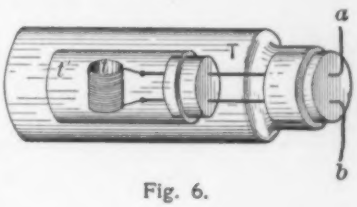


Fig. 6.

Since both the mercury and the wire were in close contact with the same good thermal conductor, and since no sudden change could affect the copper tube by virtue of the double air space, the wire was *very* nearly at the temperature of the mercury. The precise moment of solidification of the mercury was told by the cessation, under slight jars, of the gliding motion peculiar to the globule so long as a surface film existed. Likewise, in passing from the solid into the liquid, at the moment a surface film was formed, the globule which in its solid state remained motionless when subjected to slight jars, would glide off on the bottom of the tube. In order to prevent the formation of white frost on the inside of the glass, which would obscure the observations, perfectly dry air was passed for a considerable length of time into the tubes, after which the bottle was sealed with paraffin around the cork. The terminals a and b were joined to the slide-wire bridge, and the bottle was cooled by surrounding it with a cotton jacket containing carbon dioxide snow. A small aperture in the jacket directly above the top of the copper tube permitted observations on the condition of the mercury globule. An artificial light could not be used, owing to low radiant efficiency and consequent emission of sufficient heat to affect the temperature of the wire. To eliminate any error that might have arisen by simply taking the resistance when the globule solidified, the latter was alternately frozen and melted many times in succession by simply pressing tightly or loosening the cotton jacket filled with CO_2

snow; and, moreover, these changes were brought about so gradually that the readings on the slide bridge corresponding respectively to the freezing and the melting of the mercury were brought within one division of each other, — where one division is equivalent to 0.2° C.

To obtain the resistance near 0°, the bottle T (Fig. 6) was surrounded by melting snow, and after the mercury of the thermometer which was inserted through the two corks, so that its bulb was inside tube t', came to a stationary point, resistance and temperature were recorded. The temperature of the copper tube did not fall quite to 0°, on account of the heat conducted in by the leading-in wires. The temperature of freezing mercury was taken as −38.6°.

Table V. gives the data from which the coefficient α was computed.

TABLE V.

Temperature.	Bridge reading.	Resistance of wire.
−38.6	402.5	7.142
+ 1.5	667.6	8.556
	$\alpha = 0.004147$	

The coefficient thus obtained lies between the determination by Dewar and Fleming, 0.00410, and that of Cailletet and Bouty, 0.00423, both being over ranges of low temperatures. From the above value of α, and from the fact that the resistance of the coil on the collar at a temperature of 19.5° was 9.461, we have

$$R_t = 8.753\,(1 + 0.004147\,t),$$

which gives temperature in terms of resistance of the collar.

(*b*) *Determination of difference of temperature of collar and bar.*

It was found by experiment that at great temperature excesses of the bar over the air, the temperature of the wire of the collar was less than the temperature of the bar directly beneath the middle turn of the coil; and as the latter temperature is that

which is required in both the statical and the absorption experiment, it was necessary to determine the correction to be applied to the temperatures obtained for the coil. This correction was arrived at under the following apparently justifiable assumption: If the collar be placed upon the bar, and the latter be heated to a certain temperature, giving a certain excess, the difference of temperature between the coil and the bar will be the same as when the bar is cooled to a temperature that gives the same excess (but of opposite sign). Three sets of observations were taken: First, when the end of the short bar containing a thermometer bulb was inserted through a board screen (viz. the end *E* of the box afterwards described, and shown in Fig. 7) and heated by an electric current to different stationary temperatures, with the collar placed upon the bar between the heated end and the thermometer; second, when the thermometer was between the heated end and the collar; and third, when the bar was heated to a uniform temperature by means of a solenoid conveying an electric current, and then allowed to cool, — in each case simultaneous readings being taken on the slide bridge to which the collar was attached, on the thermometer whose bulb was inside the metal bar, and on the thermometer which registered the temperature of the surrounding medium. From the data thus obtained, two curves were plotted with abscissæ as temperature excess of collar over air, and ordinates as temperature difference of bar and collar. The first curve was drawn from the first two sets of observations, and was used in correcting temperatures obtained from the resistance of the collar in the statical experiment; the second curve was drawn from the last set of observations, and gave the correction in case of the curve of absorption. Although the points located were quite irregularly disposed, they indicated the direction of the curve, and showed that the correction is by no means inappreciable, being at a temperature excess of 80°, between 4° and 5°.

II.

Determination of Data for the Statical Curve.

We now come to the determination of the final distribution of temperature along the bar when one end is maintained at the constant temperature of about $-70°$ C. When high temperatures are used, this experiment offers no special difficulties; but as soon as the bar becomes very cold, frost will be deposited upon it, which not only vitiates the surface, but prevents the movement of the collar. The bar must therefore be placed in an air-tight box with everything so arranged that the dew-point of the air within it may be reduced to a very low temperature, that the collar may be moved along the bar without introducing air into the box, and lastly, that a *constant* low temperature may be maintained at one end of the bar.

The box was constructed of dry wood with glass sides, and to insure imperviousness to moisture, the wood was painted on the inside with shellac and twice with asphalt varnish. The complete box is shown in Fig. 7. The front and back sides are each formed

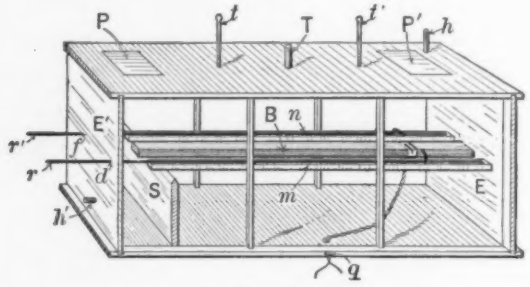


Fig. 7.

of three panes of glass. P and P' are two movable glass plates that were sealed during the experiment. B is the copper bar, one of whose ends projects about 1 cm. through the wooden end E of the box, while the other end rests on the wooden support S. r and r' are two iron rods about 8 mm. in diameter, which pass through the end E' of the box on a level with the bar, and which admit of longitudinal movement in the grooves of two wooden

supports, *m* and *n*. The inside ends of these rods are so bent that a small torsional force applied at their protruding extremities will bring these ends in contact with the projections *c* or *c'* (Fig. 1, Part I.) of the collar. A longitudinal movement of the rods will then move the collar to any desired division on the bar, without the introduction of air, inasmuch as they pass through tightly fitting rubber tubes at *d* and *f*. The terminals of the collar were joined to a cable, and the latter was brought out of the box at *q*. A copper test tube *T*, extending into the box about 5 cm., provided means of testing the dew-point of the air within by showing whether frost was deposited on its surface when it was filled with a cooling mixture of CO_2 snow and ether. *t* and *t'* are thermometers used to determine the temperature of the air in the box. For about 24 hours immediately preceding the time of observations, air which had previously traversed a series of Y-shaped tubes filled with pumice-stone saturated with concentrated H_2SO_4, was forced into the box through *h*, and after diffusing itself through the air in the box, found its exit at *h'*. In this way the dew-point was sufficiently reduced to prevent deposition of frost on the tube *T* when filled with CO_2 snow and ether.

The end of the bar that extended through the wooden box was neatly soldered into the side of a copper box which had a capacity of about a liter. Fig. 8 is a horizontal cross-section of the cooled end of the bar and its surroundings. Around the copper box *B*, into which the cooling mixture was placed, were three wooden boxes *b*, *b'*, *b''*, having only one common side, and separated from each other by the three air spaces *a*, *a'*, *a''*, loosely filled with cotton to prevent convection currents. These boxes were provided with small slide covers for the purpose of renewing the liquid in the copper receptacle.

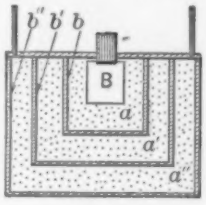


Fig. 8.

When the test for the dew-point showed that the air was sufficiently dry to begin cooling the bar, the box at the end of the bar was filled with ether, and CO_2 snow added until the liquid became saturated with the snow. The temperature of this mixture, according to Cailletet and Colardeau, is $-75°$; but since its exact

temperature was not required in the experiment, no determination of it was made. According to the authority above quoted, this mixture of a *saturated* solution of carbon dioxide snow in ether undergoes scarcely any changes of temperature. The dioxide was obtained in this solid state by drawing off the liquid kept in iron cylinders under high pressure. About two quarts of snow were drawn off at each time, and this was maintained ready for use with small loss from evaporation, in a box surrounded with other boxes, much in the manner as the liquid at the end of the bar was guarded from heat.

To test the constancy of the temperature at the end of the bar, two devices were employed. A thermo-junction was placed in the

TABLE VI.

DATA FOR CURVE OF FINAL DISTRIBUTION OF TEMPERATURE ALONG THE BAR WHEN ONE END IS MAINTAINED AT A CONSTANT LOW TEMPERATURE.

Dist. in cm. from cold end.	Temp. of air in box.	Bridge readings.	Resistance from curve.	Resistance of collar.	Temp. of collar	Excess collar over air.	Excess bar over collar.	Excess bar over air.
0	14.7	378.6	7.550	6.900	−51.1	−65.7	−3.3	−69.0
5	14.6	417.5	7.751	7.900	−45.6	−60.2	−2.8	−63.0
10	14.8	453.4	7.936	7.286	−40.4	−55.2	−2.4	−57.6
15	14.9	477.6	8.061	7.411	−37.0	−51.9	−2.0	−53.9
20	15.0	498.2	8.167	7.117	−34.1	−49.1	−1.9	−51.0
25	15.1	518.4	8.272	7.662	−31.2	−46.3	−1.7	−48.0
30	15.2	543.0	8.400	7.750	−27.6	−42.9	−1.4	−44.3
35	15.4	555.1	8.466	7.816	−25.8	−41.2	−1.3	−42.5
40	15.6	570.8	8.556	7.906	−23.3	−38.9	−1.2	−40.1
45	15.7	582.7	8.627	7.977	−21.4	−37.1	−1.0	−38.1
50	15.8	594.7	8.701	8.051	−19.3	−35.1	−0.9	−36.0
55	15.9	604.1	8.753	8.103	−17.9	−33.8	−0.9	−34.7
60	15.9	611.7	8.793	8.143	−16.8	−32.7	−0.8	−33.5
65	15.9	620.5	8.833	8 183	−15.7	−31.6	−0.8	−32.4
70	15.7	627.7	8.866	8.216	−14.8	−30.5	−0.7	−31.2
75	15.7	632.3	8.888	8.238	−14.2	−29.9	−0.7	−30.6
80	15.6	637.6	8.915	8.265	−13.4	−29.0	−0.6	−29.6
85	15.9	643.7	8.948	8.298	−12.5	−28.4	−0.6	−29.0
90	15.2	639.8	8.928	8.278	−13.1	−28.3	−0.6	−28.9
95	14.9	643.3	8.947	8.297	−12.6	−27.5	−0.6	−28.1
100	15.3	646.0	8.962	8.312	−12.2	−27.5	−0.6	−28.1

cooling mixture very close to the end of the bar, and the terminals joined through keys to the slide bridge galvanometer. Also, just inside the end E of the wooden box, a few turns of the 0.002 in. insulated wire were wound directly on the bar, and its terminals joined to a Wheatstone plug bridge.

About three hours after the cooling mixture was placed in the receptacle B (Fig. 8), the bar came to a statical temperature condition, as was indicated by the slide bridge to which the collar was connected. By frequent observations of the thermo-element, and of the resistance of the fine wire directly on the bar, it was ascer-

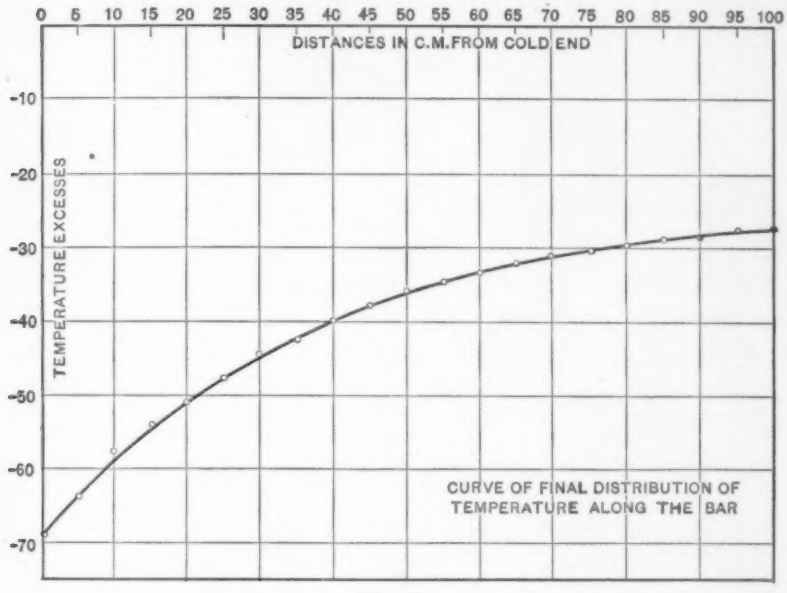

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
DISTANCES IN C.M. FROM COLD END
-10
-20
-30
-40
-50
-60
-70
TEMPERATURE EXCESSES
CURVE OF FINAL DISTRIBUTION OF TEMPERATURE ALONG THE BAR

Fig. 9.

tained that the fluctuations of temperature at the end of the bar were not large enough to appreciably affect the distribution of temperature along the bar. In Table VI. are the observed and calculated data for the construction of the statical curve shown in Fig. 9. The second column is the average of the readings of the two thermometers that registered the temperature inside the box. Column 4 is obtained from the calibration curve of the slide bridge. Column 5, which is made up from the preceding column

by subtracting the resistance 0.65 of the line in the room, and not subjected to temperature changes, is converted into temperatures of column 6 by the formula already obtained,

$$R_t = 8.753\,(1 + 0.004147\,t).$$

Subtracting (algebraically) the temperature of the air from numbers in column 6, gives column 7, and adding to these the temperature difference of collar and bar as obtained by curve already described, gives the last column, which, together with the corresponding distances in centimeters of column 1, gives the coördinates of the curve in Fig. 9.

III.

Cooling and Absorption Experiments.

In investigations on conductivity, it seems to have been customary to obtain the curve of cooling of a short bar of the same material and cross-section, and to use this curve in connection with the statical curve of the long bar. In the present case, experiments of cooling on the two bars, whose lengths it will be remembered were respectively about 100 cm. and 20 cm., showed that their rates of cooling at the same temperature excess were noticeably different, the surface being as nearly the same as possible. And since it is probable that the emissivity is a function of the temperature, it was necessary to obtain the rates of heat absorption of the long bar at different negative excesses. Although the method devised for obtaining the requisite observations for the absorption curve of the short bar was applicable to the long one, it was not feasible on account of the limited supply of liquid CO_2 at our disposal, and hence the rates of absorption of the long bar were obtained in an indirect manner. Now it must be nearly, if not rigorously, true that the ratio of the rates of cooling of the short and long bars at any positive excess is the same as the ratio of their rates of heat absorption at an equal negative excess, — other conditions being the same. Hence if at a certain excess, B and b are the tangents to the curve of cooling of the long and

short bars respectively, and A and a are the respective tangents to the curves of absorption at an equal negative excess, we have

$$A=\frac{aB}{b}, \tag{2}$$

which gives the rates of heat absorption of the long bar in terms of obtainable quantities. Observations must hence be taken for three curves, viz., absorption curve of short bar and the cooling curves of both bars.

(*a*) *Absorption curve of short bar.*

In order to obtain observations for this curve, the bar must be cooled in a perfectly dry atmosphere, to a uniform temperature as low as the lowest temperature observed in the statical experiment; it must be removed from the cooling apparatus and placed in the *dry* atmosphere of the wooden box, so as to be under conditions similar to those of the long bar in the statical experiment; and, moreover, this must be done with the collar on the bar, not only to afford a means of ascertaining its temperature condition before being removed from the cooling chamber, but to be ready to begin observations soon after thus removed.

The above requirements were met in the following way: The collar was placed on the short bar and the latter placed in a specially constructed, tightly fitting, tin box, which is shown in Fig. 10. This box, B', which contained the bar, was soldered into another tin box, B, such that there was an air space of about 2 cm. surrounding the box B' on five sides. In the figure a portion of the outside box is broken away, showing this air chamber, with which there was no communication except through the two glass tubes t and t', each about $\frac{3}{4}$ inch in diameter, that extended through the end E of the wooden box of

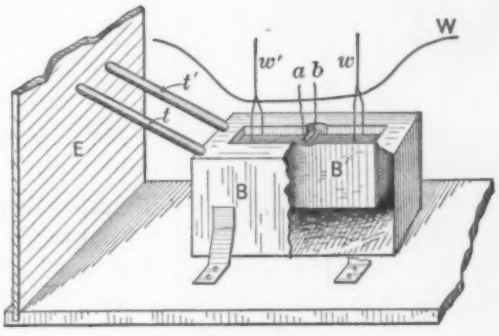


Fig. 10.

Fig. 7, from which the long bar had been removed. *a*, *b* are the terminals of the collar, and *w*, *w'* are wires passing through very

TABLE VII.

DATA FOR TIME CURVE OF ABSORPTION OF SHORT BAR.

Time.	Temp. of air.	Bridge readings.	Resistance from curve.	Resistance of collar.	Temp. of collar.	Excess collar over air.	Excess bar over collar.	Excess bar over air.
min. sec.								
0 15	16.1	371.6	7.515	6.865	−52.0	−68.1	−4.0	−72.1
1 5	16.1	383.3	7.574	6.924	−50.4	−66.5	−3.8	−70.3
2 24	16.1	400.9	7.665	7.015	−47.9	−64.0	−3.5	−67.5
4 12	16.0	422.7	7.778	7.128	−44.8	−60.8	−3.3	−64.1
5 16	16.0	435.7	7.845	7.195	−42.9	−58.9	−3.1	−62.0
6 52	16.0	452.7	7.933	7.283	−40.5	−56.5	−2.8	−59.3
8 30	16.0	470.2	8.023	7.373	−38.0	−54.0	−2.6	−56.6
9 55	16.0	484.8	8.099	7.449	−35.9	−51.9	−2.4	−54.3
10 58	15.9	494.9	8.150	7.500	−34.5	−50.4	−2.3	−52.7
12 47	15.9	512.0	8.240	7.590	−32.0	−47.9	−2.1	−50.0
13 46	15.9	520.5	8.284	7.634	−30.8	−46.7	−2.0	−48.7
14 59	15.9	530.4	8.334	7.684	−29.5	−45.4	−1.8	−47.2
16 19	15.9	542.0	8.396	7.746	−27.8	−43.7	−1.7	−45.4
18 13	15.9	556.5	8.474	7.824	−25.6	−41.5	−1.5	−43.0
21 2	15.8	576.0	8.587	7.937	−22.5	−38.3	−1.4	−39.7
23 19	15.8	590.2	8.675	8.025	−20.0	−35.8	−1.2	−37.0
25 33	15.9	603.1	8.748	8.098	−18.0	−33.9	−1.1	−35.0
28 16	15.9	617.3	8.817	8.167	−16.1	−32.0	−1.0	−33.0
30 38	15.9	628.1	8.867	8.217	−14.7	−30.6	− .9	−31.5
32 18	15.9	635.3	8.903	8.253	−13.7	−29.6	− .9	−30.5
35 0	15.9	646.1	8.963	8.313	−12.1	−28.0	− .7	−28.7
37 13	15.9	654.7	9.010	8.360	−10.8	−26.7	− .7	−27.4
39 8	15.9	661.4	9.051	8.401	− 9.7	−25.6	− .7	−26.3
40 54	15.9	667.5	9.088	8.438	− 8.7	−24.6	− .6	−25.2
42 34	16.0	673.2	9.123	8.473	− 7.7	−23.7	− .5	−24.2

small holes in the top of the wooden box and fastened respectively to two loops of small wire that passed around each end of the bar.

In order to cool the bar, a viscous mixture of CO_2 snow and ether was poured into one of the glass tubes till the liquid began to rise in the tubes, which easily carried off the vapor arising from

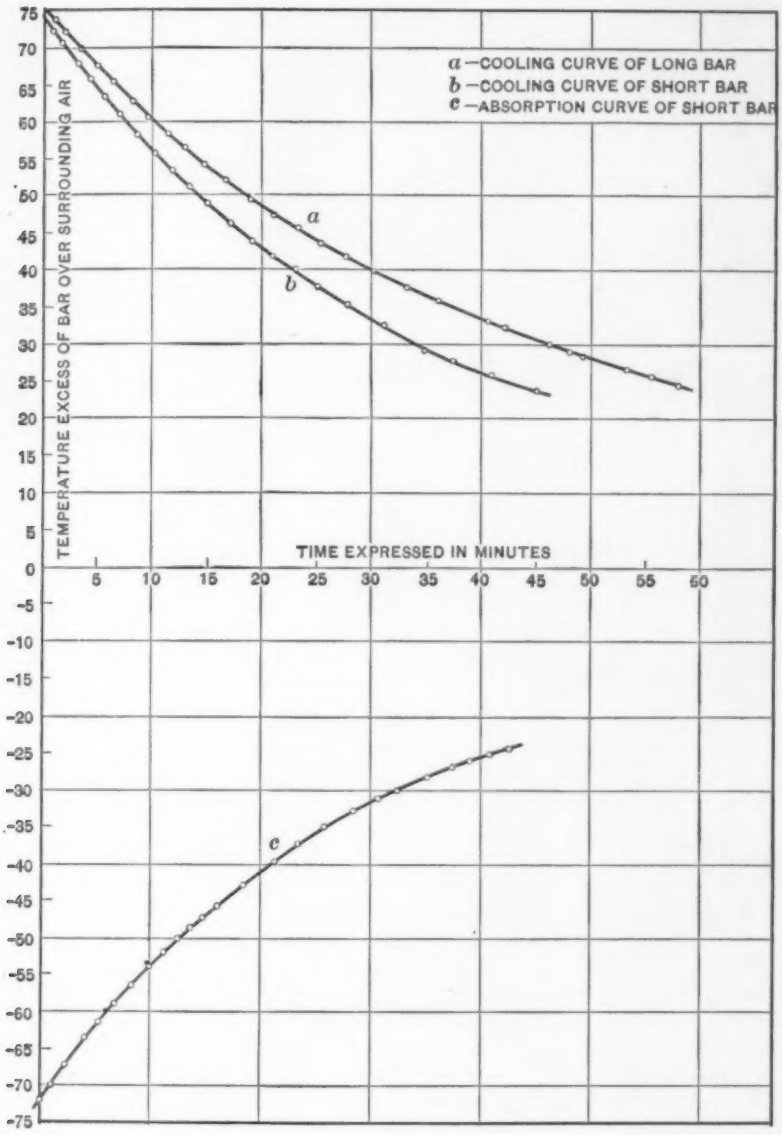


Fig. 11.

the violent ebullition of the liquid. To maintain the liquid at a low temperature, carbon dioxide snow was constantly pushed down the glass tubes (alternately) by means of a closely fitting

TABLE VIII.

DATA FOR CURVES OF COOLING.

Short bar.			Long bar.		
Time.		Excess of bar over air.	Time.		Excess of bar over air.
min.	sec.		min.	sec.	
0	40	73.2	0	36	74.2
1	53	70.7	1	57	71.7
3	12	68.2	3	27	69.4
4	27	65.7	4	51	67.2
5	49	63.3	6	15	65.0
7	11	60.9	7	56	62.7
8	40	58.3	9	35	60.4
10	10	55.9	11	18	58.3
11	50	53.5	13	0	56.1
13	25	51.2	14	53	54.0
15	8	48.9	16	45	51.9
17	8	46.8	18	45	49.8
19	12	44.5	20	50	47.7
21	14	42.2	23	15	45.7
23	21	40.0	25	26	43.7
25	40	37.6	27	57	41.8
28	11	35.3	30	21	39.9
31	15	33.1	33	15	38.1
35	25	29.6	36	8	36.0
37	18	28.6	40	40	33.1
40	55	26.6	42	27	32.3
44	45	24.3	45	48	30.4
			47	43	29.3
			49	38	28.5
			53	31	26.8
			55	41	25.8
			57	53	24.8

wooden piston. When the slide bridge to which the collar was attached showed that the temperature was sufficiently low and constant, the bar was lifted out of the receptacle by means of wires w, w' (Fig. 10), and swung off out of the influence of the

cooling chamber by aid of the wire *W* which passed through each end of the large wooden box. Soon after, observations were begun. The air of the large wooden box had, of course, been previously thoroughly dried in the manner already described.

In Table VII. are the data, both observed and calculated, for the absorption curve of short bar. As the columns are arranged in the same manner as in Table VI., no explanation is necessary. From the first and last columns of this table, curve *c*, Fig. 11, was constructed.

(*b*) *Cooling curves of both bars.*

These observations were made according to the method which has already been described in IV., Part I., except that in this case each bar was allowed to cool within the wooden box, in order to preserve the conditions for radiation as nearly the same as possible as those for absorption. The bridge was calibrated directly for temperatures, as was described in Part I. The reason for calibrating the bridge for these observations directly for temperatures of the bar is twofold: first, in order to obtain readings on the slide bridge over the range of 20° to 100°, the resistances r and r' (Fig. 2, Part I.) had to be changed, thus, in any case, necessitating a re-calibration of the bridge; second, the temperature coefficient of the coil on the collar might be slightly different for high temperatures, and this difference would have to be determined.

In Table VIII. are given the data for the curves of cooling of both bars, from which curves *a* and *b* (Fig. 11) were constructed. The difference in the rates of cooling of the long and short bars is plainly evident, and also a slight difference in the direction of the curves of cooling and absorption of the short bar.

IV.

Deduction of Results.

It now remains to construct the curve whose area, multiplied by the product of specific heat and density, is equal to the quantity of heat absorbed per second by the bar when one of its ends was main-

tained at a low temperature. Table IX. contains the calculated data for this curve. Columns 2, 3, and 4 are the respective tangents to the cooling curve of short bar, absorption curve of short bar, and cooling curve of long bar. These were taken from the

TABLE IX.

DATA FOR CURVE OF RATES OF ABSORPTION CORRESPONDING TO DIFFERENT POINTS ALONG THE BAR WHEN UNDER THE CONDITIONS OF THE STATICAL EXPERIMENT—INTEGRATION CURVE.

Distance in cm. from cold end.	Tangent to curve of cooling of short bar. $\left(-\frac{dv}{dt}\right)$	Tangent to curve of absorption of short bar. $\left(\frac{dv}{dt}\right)$	Tangent to curve of cooling of long bar. $\left(-\frac{dv}{dt}\right)$	Tangent to curve of absorption of long bar (calculated). $\left(\frac{dv}{dt}\right)$
0	0.03355	0.03500	0.02721	0.02840
5	0.02950	0.03100	0.02400	0.02522
10	0.02773	0.02872	0.02166	0.02240
15	0.02375	0.02545	0.01914	0.02050
20	0.02180	0.02330	0.01720	0.01838
25	0.02010	0.02240	0.01548	0.01724
30	0.01890	0.02125	0.01400	0.01574
35	0.01810	0.02030	0.01300	0.01460
40	0.01700	0.01880	0.01214	0.01343
45	0.01594	0.01695	0.01146	0.01220
50	0.01533	0.01592	0.01080	0.01123
55	0.01421	0.01392	0.01030	0.01010
60	0.01328	0.01236	0.01000	0.00931
65	0.01267	0.01160	0.00944	0.00865
70	0.01200	0.01097	0.00900	0.00823
75	0.01160	0.01067	0.00853	0.00785
80	0.01110	0.01037	0.00827	0.00772
85	0.01076	0.01033	0.00796	0.00764
90	0.01053	0.01024	0.00783	0.00762
95	0.01046	0.01010	0.00780	0.00753
100	0.01024	0.00977	0.00773	0.00752

curves of Fig. 11 at temperature excesses of the bar, which were read from the statical curve of Fig. 9 at the different distances of column 1. The numbers of the last column are tangents to the hypothetical absorption curve of long bar, and are obtained according to formula (2); *i.e.*, by dividing the product of columns 3 and

4 by column 2. From the first and last columns the curve in Fig. 12 was plotted, and its area was obtained by integration in

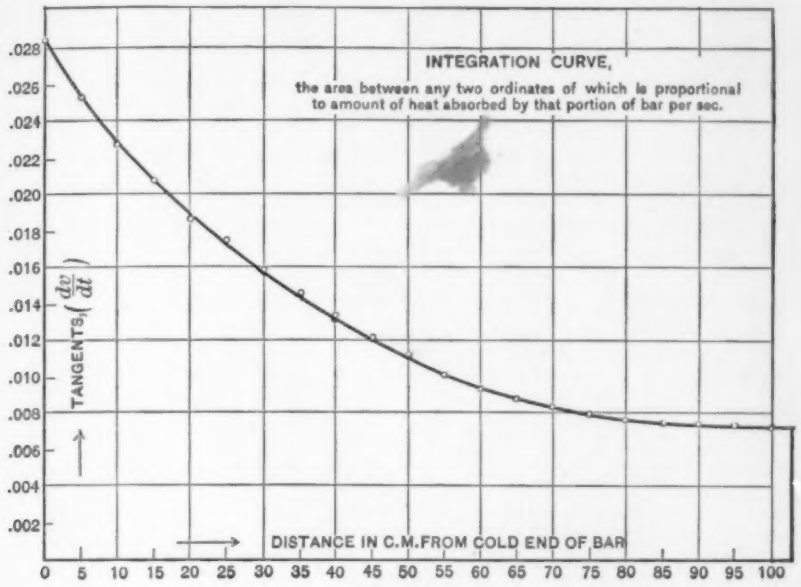


Fig. 12.

TABLE X.

DERIVED DATA FOR THE CURVE OF CONDUCTIVITY (K).

Distance (x) in cm. from cooled end of bar.	area, $\int_x^a \frac{dv}{dt}\,dx$.	Tangent to statical curve. $\frac{dv}{dx}$.	Temperature of copper.	Conductivity, K.
80	0.1692	0.125	−13.6	1.059
65	0.2917	0.217	−16.3	1.050
55	0.3855	0.289	−19.0	1.040
45	0.4952	0.372	−22.4	1.035
40	0.5575	0.422	−24.5	1.025
35	0.6255	0.475	−26.9	1.020
30	0.6999	0.542	−29.6	0.999
25	0.7812	0.605	−32.6	0.996
20	0.8701	0.690	−36.0	0.971
15	0.9676	0.785	−39.8	0.946
10	1.0748	0.873	−44.0	0.942
5	1.1939	0.986	−48.7	0.923
0	1.3273	1.095	−53.9	0.921

parts as is indicated in column 2 of Table X. In this column, to each number is added the area corresponding to the heat absorbed per second by the free end of the bar. This table is made up in the same way as the corresponding table of Part I. For the want of experimental data on the relation between specific heat and temperature over the range 0° to −60°, the same formula was used here as in the case of high temperatures, viz. :—

$$S_t = .0892(1 + 0.00073\, t).$$

Also the same variation in density was made use of, viz.:—

$$D_t = 8.862(1 - 0.000056\, t).^1$$

In Fig. 13 is the curve of conductivity plotted from the last two columns of Table X. and Table IV. (Part I.), and shows graphically the results of the complete investigation.

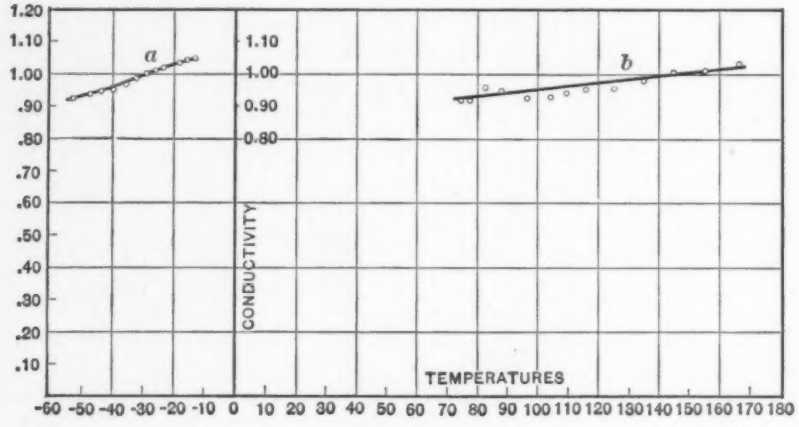


Fig. 13.

V.

Discussion of Results.

It is to be observed that the average value obtained for K over the range −54° to −13° is slightly higher than the average value over the range 74° to 166°, — being in the former case 0.994, and in the latter, 0.954. A curious fact is the much greater slope of

[1] See V., Part I.

the curve over low temperatures. From these results one of two inferences may be drawn: first, that a maximum value of K exists at a temperature near 0°, since, if the curves a and b were joined, a point of inflexion must exist between $-10°$ and $+70°$; second, that owing to some error, the values of K in a are too great, or that the values of K in b are too small, thus producing the irregularity in the curve. As regards the latter inference, the authors state that the observations were not only taken with the greatest of care, but were also reduced with the utmost precaution to eliminate error. Of course, if values of K were obtained for temperatures between $-10°$ and $+70°$, a more definite conclusion could be reached. The reason those points were not obtained, thus filling in between the curves a and b (Fig. 13), may be briefly stated thus: when the excess of the temperature of the bar over air, or air over bar, is 30° or less, the slope of the curve of final distribution of temperature along the bar is so small in the case of copper, that great inaccuracies arise in the calculation of the tangents. A method for overcoming this difficulty, but which would impose conditions hard to realize, is to place the bar in an atmosphere whose temperature is maintained constant at 0°, and a temperature of about 80° kept at one end; or to maintain a temperature of about $-15°$ at one end of the bar which is placed in an atmosphere kept constant near 80°. In either case a good gradient would be obtained.

Over both ranges of temperature the results show an increase of K with increase of temperature. This is in accordance with the results of Tait, Lorenz, and Kirchoff and Hansemann, but is contrary to those of R. W. Stewart, who found a decrease of K in electrolytic copper, with increase of temperature over a range of high temperatures. His values of K were considerably greater than those hitherto obtained, a fact indicating that conductivity increases with the purity of the metal. Stewart's value of K at 0° obtained by extrapolation, coincides almost exactly with the K_0 obtained from curve a. It is to be hoped that further investigation will be made on electrolytic copper at temperatures near 0°, also at low temperatures, by using, perhaps, a cooling mixture

whose boiling point is even lower than that of carbon dioxide snow and ether.

The authors desire to acknowledge their obligations to Dr. E. L. Nichols, who not only suggested the line of work, but who made many valuable suggestions as to details throughout the investigation.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
December, 1894.

ON TERNARY MIXTURES.

FIRST PAPER.[1]

By Wilder D. Bancroft.

FOLLOWING out the analogy between dissolved substances and gases, Nernst deduces the law that, when two dissolved substances have no common ion and do not react chemically, the influence of each on the solubility of the other is zero, within certain undefined limits. He says:[2] "Die Analogie zwischen der Auflösung und Sublimation bezw. Dissociation fester Stoffe zeigt sich nun auch deutlich ausgesprochen, was den Einfluss fremden Zusatzes betrifft. Ebenso wenig wie die Sublimationsspannung bei Gegenwart fremder indifferenter Gase sich ändert, wird die Löslichkeit eines festen Stoffes durch Zusatz eines zweiten (in nicht zu grosser Menge) beeinflusst, wofern der hinzugefügte fremde Stoff nicht chemisch auf jenen einwirkt; und ebenso wie die Dissociationsspannung im höchsten Maasse durch Zusatz eines der gasförmigen Zersetzungsproducte beeinflusst wird, so variirt entsprechend auch die Löslichkeit derjenigen Stoffe, bei welchen die Auflösung mit einem mehr oder weniger vollständigen Zerfall verbunden ist, die also bei ihrer Auflösung mehrere Molekülgattungen liefern, wenn eine dieser letzteren der Lösung hinzugefügt wird." There are several things in this statement which are open to criticism. If taken literally, the author implies a fundamental difference between solutions of liquids in liquids, and solids in liquids, a distinction which is not in accordance with the view that in dilute solutions the solute,[3] whether liquid or solid in the pure state, behaves like a gas at that temperature.

[1] A paper presented to the American Academy of Arts and Sciences, May 9, 1894.

[2] Theor. Chemie, p. 383.

[3] There seems to me a need for a word denoting the dissolved substance. In future I shall use the word "solute," meaning the substance dissolved in the solvent. Instead of the phrase "infinitely miscible liquids," I propose "consolute" liquids.

If applied to any dissolved substance, the statement just quoted is too inaccurate to need any comment. The precipitation of salts by alcohol is a well known instance where it does not apply, and, in general, adding to a solution a substance in which the solute is practically insoluble diminishes the solubility of the latter. This is recognized by Nernst, for he has based a method for determining reacting weights upon it.[1] Even if limited to solids, the proposition cannot be admitted. We have the precipitation of lactones by potassium carbonate as an intermediate step, and the precipitation of salts by phenol as a definite case of diminished solubility without the presence of a common ion. Other cases could be cited, if necessary, and there are also examples where an increase of solubility takes place when a solid substance is added to a solution containing another solid as solute. The explanation usually offered under these circumstances is, that "double molecules" are formed, a mode of getting round the facts which is not always entirely satisfactory.

Since in the application of the gas laws to solutions there has been observed no difference between a solid and a liquid when dissolved, I am inclined to think that the general statement should be, that in all cases where a third substance, *B*, is added to a solution of *A* in *S*, the solubility of *A* undergoes a change. This variation may be large or small, positive or negative, depending on the nature of the three substances, *A*, *B*, and *S*. When both *A* and *B* are liquids, or even when only one of them is, the effect is so marked as to be familiar to all; when both are solids, the effect is not yet recognized by so competent an authority as Nernst.

The work of the last few years on solutions has been devoted to bringing out the analogy between the dissolved substance and gases. In the cases of changed solubility, no common ion being present, the analogy is no longer with gases, but with liquids. The added substance acts as a liquid, precipitating the solute more or less in proportion as the dissolved substance happens to be more or less soluble in it. The laws governing these displacements are entirely unknown, with the exception of Nernst's Dis-

[1] Zeitschr. f. ph. Chem., VI. 16. 1890.

tribution Law,[1] which is only a first approximation, in that it takes no account of the changing mutual solubilities of the hypothetically non-miscible liquids. Under these circumstances it seemed to me desirable to investigate the laws governing systems composed of three substances, and the experiments which I communicate in this paper have been made on the simplest form of ternary mixtures, — that where all three substances are liquids. The subject has been very little studied, the only researches known to me being by Tuchschmidt and Follenius,[2] Berthelot and Jungfleisch,[3] Duclaux,[4] Nernst,[5] and Pfeiffer.[6] Of these, all except the first and last deal with the equilibrium between two liquid phases; the paper of Tuchschmidt and Follenius contains but one series of measurements, while Pfeiffer remarks, apropos of his own extended investigations, that "there is very little to be made out of them." In this he does himself an injustice, for, as I shall show, his results are very satisfactory and astonishingly accurate when one remembers how they were made.

The simplest case of three-liquid systems is when one has two practically non-miscible liquids, and a third with which each of the others is miscible in all proportions; for then any complication due to the mutual solubility of the two dissolved liquids is avoided. It is possible to say something *a priori* about the law which governs these saturated solutions. Let A and B be two non-miscible liquids, S the common solvent with which A and B are miscible in all proportions when taken singly, and let the quantity of S remain constant, so that we are considering the amounts of A and B, namely x and y, which will dissolve simultaneously in a fixed amount of S. It is known, experimentally, that the presence of A decreases the solubility of B, and *vice versa*; it is required to find the law governing this change of solubility. This, being a case of equilibrium, must come under the general equation of equilibrium.

$$(1) \qquad \frac{\delta F(x, y)}{\delta x} dx + \frac{\delta F(x, y)}{\delta y} dy = 0,$$

[1] Teilungssatz.
[2] B. B., IV. 583. 1871.
[3] Ann. chim. phys., [4.], XXVI. 396. 1872.
[4] Ibid., [5.], p. 264. 1876.
[5] Zeitschr. f. ph. Chem., VI. 16. 1890.
[6] Ibid., IX. 469. 1892.

where dx and dy denote the changes in the concentrations of A and B, respectively.

This equation, though absolutely accurate, is of no value practically so long as the differential coefficients are unknown functions. In regard to them we may make two assumptions. The decrease in the solubility of A may be proportional to the amount of B added, and independent of the amounts of A and B already present in the solution. The differential equation expressing this is:—

$$adx+bdy=0, \tag{2}$$

where a and b are proportionality factors and constants. This equation may be rejected on *a priori* grounds, because it does not show that when B is absent, the miscibility of A with S is infinite, and also because it has no similarity with the other equations representing chemical equilibrium. The second assumption is that the change in solubility may be a function of the amounts of A and B already present. This is the usual condition of chemical equilibrium, and is known as the Mass Law. Its mathematical expression is:—

$$\frac{\alpha dx}{x}+\frac{\beta dy}{y}=0, \text{ or}$$

$$\alpha d\log x+\beta d\log y=0, \tag{3}$$

where x and y denote the amounts of A and B in a constant quantity of S, α and β are proportionality factors, and the logarithms are natural logarithms.

If α and β are constants, this equation is integrable, and gives, when cleared of logarithms:—

$$x^{\alpha}y^{\beta}=\text{Constant}. \tag{4}$$

If we make $\frac{\alpha}{\beta}=n$, we shall have:—

$$x^{n}y=C, \tag{5}$$

where C is of course different in value from the constant in equation (4).

Before we proceed to test equation (5) experimentally, it remains to be seen in what unit x and y should be expressed. It is obvious

that the nature of the unit has no effect on the general form of the equation, nor upon the exponential factor n. The only change will be in the value of the integration constant log C, so that the measurements may be expressed in any form that is convenient, as chemical units,[1] for example, grams per liter, volumes, reacting volumes, or anything else. It is not even necessary that x and y be expressed in the same unit, though it would probably always be more practical. In my own experiments, x and y are expressed in cubic centimeters because they were measured directly as such, and in this way it was not necessary to make determinations of the densities of the liquids used, nor any assumptions in regard to their reacting weights. Equation (5) will not remain unchanged if the reacting weight of A or B varies, that is, if the ratio of the active mass to the actual mass changes as x or y changes. The converse of this is also true, that if the system follows the law $x^n y = C$, the common solvent remaining constant, the reacting weights of the substances A and B cannot have varied with the concentration.

I have found that the equation, $x^\alpha y^\beta = \text{Constant}$, is the expression representing the saturated solutions of two non-miscible liquids in a constant quantity of a consolute liquid. I find, however, that in most cases the concentrations cannot be given by one curve, but involve two, so that for one set of concentrations I have the relation $x^{n_1} y = C_1$, for the other set $x^{n_2} y = C_2$. This cannot be true unless the two sets of saturated solutions correspond to different conditions. This is the case. Duclaux[2] found that a saturated solution of amylalcohol and water in ethylalcohol became turbid on adding a drop either of amylalcohol or of water. In other words, the solution was sensitive to an excess of either liquid.[3] I have confirmed this result, and it is perfectly general. It is not proper,

[1] I have adopted the following nomenclature for molecular and atomic weights, viz. reacting and combining weights. As the reacting weight is proportional to the chemical unit experimentally, I propose that the gram molecule in the unit of volume (reacting weight in grams per liter) be called the chemical unit, or simply the unit. The object of these arbitrary changes in our chemical terms is to do away with everything involving or implying the assumption of the existence of molecules and atoms.

[2] Ann. chim. phys, [5.], VII. 264. 1876.

[3] Ostwald, Lehrbuch, I. 819.

however, to draw the conclusion that the solution is saturated in respect to both liquids. If to a given saturated solution of chloroform, water, and alcohol, for instance, one adds a drop of water or of chloroform, the solution becomes turbid; but what separates out is the same in both cases. It is analogous to a saturated solution of salt in a mixture of alcohol and water. It is indifferent whether one adds alcohol or salt to the solution. In either case, there is a precipitate; but in both cases the precipitate is salt, and the solution is saturated in respect to salt, not in respect to alcohol. It is not so easy to see what takes place in a system composed of liquids because the precipitate, being itself a liquid, dissolves part of the solution, and the new phase is not composed of pure substance. This need not trouble us, for, theoretically at any rate, the precipitate may be treated as pure liquid, and the final equilibrium looked upon as due to a subsequent reaction. One of the two curves represents, then, the set of solutions which is saturated in respect to chloroform, and not in respect to water. Whether one adds water or chloroform to these solutions, the precipitate is chloroform. The other curve represents the mixtures which are saturated in respect to water, and not in respect to chloroform. Either water or chloroform, when added to these solutions, produces a precipitate of water. These two sets of solutions are easily distinguishable qualitatively, because in the first case the new phase, containing a large percentage of chloroform, is denser than the mixture from which it separates, while in the second case the new phase, containing chiefly water, is lighter than the original solution. The point where the new phase changes from being denser to being lighter than the first phase is the point of intersection of the two curves. At this point only is the nature of the precipitate determined by the nature of the infinitely small excess added. The intersecting point represents the concentration at which, were chloroform and water solids at that temperature, both could be in equilibrium with the solution and its saturated vapor. It corresponds to the concentration of a solution containing two salts with a common ion which is in equilibrium with the two solid salts, formation of a double salt being excluded. In one respect the analogy between a system having three liquid components and

one composed of two solids and a liquid does not hold. If to a saturated solution of silver bromate silver acetate is added, the precipitate is silver bromate, and, conversely, the precipitate is silver acetate if silver bromate be added to a saturated solution of silver acetate. The salt with the less concentration precipitates the one with the greater, up to a certain point. In a chloroform-water-alcohol mixture in which chloroform is present in large quantities, the precipitate is water, or the substance with the greater precipitates the one with the lesser concentration. This difference of behavior is due to the new phase being a solid in the one case and a liquid in the other. By a suitable choice of the three components, and by varying the temperature, the substance in respect to which the solution was saturated could be made to separate either as a liquid or a solid phase, and this difference could be made zero. The transition point would come when the equilibrium was between four phases, one solid, two liquid, and one gaseous.

There is no apparent theoretical reason why the two curves should not be prolonged beyond their intersection; but there is a very good practical one. Beyond the point of intersection the curves denote saturated but labile solutions, and a supersaturated system composed of liquids is almost impossible to realize. When I come to the study of ternary mixtures having one or more solid components, I hope to be able to follow one of the curves at least beyond the intersecting point; but in the present work I have made no such attempt.

I will now describe the method used in my work, and then take up the experimental data obtained. As pairs of non-miscible liquids, I have taken chloroform and water, benzol and water; and as consolute liquids, ethylalcohol, methylalcohol, and acetone. The next point was how to determine the composition of the saturated solutions. The methods of quantitative analysis are useless in this case; but the problem is solved without difficulty by quantitative synthesis. Instead of making a saturated solution and analyzing it, I measured the quantities required to make a saturated solution at the required temperature. Definite amounts of the consolute liquids were put in test tubes by means of a carefully graduated pipette; varying quantities of one of the

non-miscible liquids were run in from a burette, and the second non-miscible liquid added from another burette to saturation. The test tubes were corked, warmed just above the temperature at which the final readings were made, so that there should be a single homogeneous liquid layer, and placed in a constant temperature bath. If the tube clouds, it is beyond the saturation point; if it remains clear, it is not up to it, the required value lying between the two. By making a series of experiments one can bring the limiting values very close together, and thus determine the saturation point with great accuracy. The constant temperature bath was at 20° C. No correction was made for the amounts of the three liquids evaporating off into the vapor space in the upper part of the test tubes; but by using different sized test tubes this space did not vary much, being about five cubic centimeters, so that the error due to this may be neglected.

The chloroform used (Squibb's) was treated with sodium bisulphite solution to free it from acetone, washed thoroughly with water, dried over calcium chloride and fractionated, twelve hundred grams going over within one quarter of a degree. Kahlbaum's crystallized benzol was recrystallized twice and fractionated to constant boiling point. The ethylalcohol was dried over lime and copper sulphate and fractionated. The lot used distilled within half a degree. Part of the acetone (from Eimer and Amend) was converted into the bisulphite compound, back again, dried over potassium carbonate and calcium chloride, and fractionated. Another portion was treated direct with calcium chloride and fractionated. I could detect no difference between the two lots. I tried to purify a sample of acetone from Cutler Brothers, purporting to be manufactured by Merck in Darmstadt; but it was so bad that I used none of it in my experiments. The methylalcohol (from Kahlbaum) was dried over anhydrous copper sulphate and fractionated.

The measurements in the tables are the mean of at least four determinations, and the error is probably not more than 5 per cent, except in the cases where the quantity of one component is less than 0.20 c.c., when it may easily rise to 10 per cent. The values for n are accurate to within 2 per cent without much question.

The values for log C are more untrustworthy, being much affected by a slight variation in n, while the term C is liable to even greater fluctuations, and is not given, as being too uncertain. Under the headings "Calc." are the values required by the formula to correspond with the experimental data for the other component. The figures in the column marked log C are Briggsian logarithms. As will be noticed, I have not always taken the mathematical mean of this column as the value of log C in the formula. It seemed better to take the value which best satisfied the experimental data, and to ignore numbers which were obviously faulty.

TABLE I.

x c.c. H_2O; y c.c. $CHCl_3$; 5 c.c. Alcohol. Temp. 20°.
Formula $x^{n_1}y = C_1$; $n_1 = 1.90$; log $C_1 = 1.190$.

Water.		$CHCl_3$.		
Calc.	Found.	Calc.	Found.	log C_1.
9.94	10.00	0.195	0.20–	1.195
8.99	9.00	0.24	0.24	1.192
7.98	8.00	0.30	0.30	1.193
7.14	7.00	0.385	0.37	1.174
6.00	6.00	0.515	0.515	1.190
5.97	5.00	0.73	0.73	1.191
3.97	4.00	1.12	1.13	1.197
	Average, ...			1.190

Formula $xy^{n_2} = C_2$; $n_2 = 1.111$; log $C_2 = 0.742$.

				log C_2.
3.00	3.00	1.73	1.73	0.741
1.99	2.00	2.49	2.51	0.745
1.01	1.00	4.66	4.60	0.737
0.92	0.91	5.07	5.00	0.736
0.755	0.76	5.96	6.00	0.745
0.635	0.63	7.06	7.00	0.738
0.55	0.55	8.00	8.00	0.743
0.48	0.49	8.86	9.00	0.750
0.43–	0.425	10.06	10.00	0.739
0.20	0.20–	20.00	20.00	0.742
0.127	0.125	30.24	30.00	0.738
	Average, ...			0.741

TABLE II.

x c.c. Water; y c.c. $CHCl_3$; 5 c.c. Methyl Alcohol. Temp. 20°.
Formula $x^{n_1}y=C_1$; $n_1=2.30$; log $C_1=1.291$.

Water.		$CHCl_3$.		
Calc.	Found.	Calc.	Found.	log C_1.
9.91	10.00	0.10	0.10	1.300
5.01	5.00	0.48	0.48	1.288
4.03	4.00	0.81	0.80	1.283
1.99	2.00	3.97	4.00	1.294
	Average, ...			1.291
Formula $x^{n_2}y=C_2$; $n_2=1.25$; log $C_2=1.061$.				
				log C_2.
1.49	1.49	7.00	7.00	1.061
1.34	1.35	7.93	8.00	1.065
1.12	1.12	10.00	10.00	1.061
	Average, ...			1.062

TABLE III.

x c.c. Water; y c.c. Chloroform; 5 c.c. Acetone. Temp. 20°.
Formula $x^{n_1}y=C_1$; $n_1=1.415$; log $C_1=0.194$.

Water.		Chloroform.		
Calc.	Found.	Calc.	Found.	log C.
5.01	5.00	0.16	0.16	0.193
4.00	4.00	0.22	0.22	0.194
3.47	3.50	0.266	0.27	0.201
3.00	3.00	0.33	0.33	0.193
2.49	2.50	0.43	0.43	0.196
2.01	2.00	0.586	0.58	0.189
	Average, ...			0.194
	1.50		0.74	
	1.20		0.83	
	1.00		0.955	
	0.93		1.00	
	0.79		1.12	
	0.71		1.20	
	0.58		1.40	
	0.53		1.50	
	0.505		1.60	
	0.38		2.00	
	0.30–		2.50	
	0.25		3.00	
	0.21		3.50	
	0.19		4.00	
	0.16		5.00	
	0.12		10.00	

Table IV.

x c.c. Water; y c.c. Benzol; 5 c.c. Alcohol. Temp. 20°.
Formula $x^n y = C$; $n = 1.60$; log $C = 0.554$.

Water.		Benzol.		
Calc.	Found.	Calc.	Found.	log C.
19.87	20.00	0.03	0.03	0.557
10.65	10.00	0.09	0.08	0.503
7.94	8.00	0.13	0.13	0.559
4.97	5.00	0.273	0.275	0.557
4.00	4.00	0.39	0.39	0.554
3.02	3.00	0.61	0.61	0.558
2.01	2.00	1.18	1.17	0.550
1.72	1.72	1.50	1.50	0.553
1.50	1.50	1.87	1.87	0.554
1.44	1.45	1.98	2.00	0.559
1.00	1.00	3.58	3.57	0.553
0.605	0.605	8.00	8.00	0.554
0.526	0.525	10.04	10.00	0.552
0.34	0.34	20.14	20.00	0.551
	Average, ...			0.551

Table V.

x c.c. Water; y c.c. Benzol; 5 c.c. Methyl Alcohol. Temp. 20°.
Formula $x^{n_1} y = C_1$; $n_1 = 1.48$; log $C_1 = 0.216$.

Water.		Benzol.		
Calc.	Found.	Calc.	Found.	log C_1.
5.05	5.00	0.15	0.15	0.211
3.95	4.00	0.21	0.215	0.223
3.01	3.00	0.32	0.32	0.211
2.00	2.00	0.59	0.59	0.216
1.40	1.40	1.00	1.00	0.216
	Average, ...			0.215
Formula $x^{n_2} y = C_2$; $n_2 = 2.00$; log $C_2 = 0.281$.				
				log C_2.
1.13	1.13	1.50	1.50	0.282
1.00	1.00	1.91	1.90	0.279
0.80	0.80	2.99	3.00	0.283
0.69	0.69	4.01	4.00	0.280
0.49	0.49	7.96	8.00	0.283
	Average, ...			0.281

TABLE VI.

x c.c. Water; y c.c. Benzol; 5 c.c. Acetone. Temp. 20°.
Formula $x^{n_1}y=C_1$; $n_1=1.40$; log $C_1=0.262$.

Water.		Benzol.		
Calc.	Found.	Calc.	Found.	log C_1.
7.97	8.00	0.10	0.10	0.264
7.00	7.00	0.12	0.12	0.262
5.04	5.00	0.19	0.19	0.258
4.03	4.00	0.26	0.26	0.258
2.99	3.00	0.393	0.395	0.264
2.49	2.50	0.51	0.51	0.265
2.18	2.20	0.61	0.615	0.269
2.01	2.00	0.69	0.69	0.260
	Average, ...			0.2625

Formula $xy^{n_2}=C_2$; $n_2=1.35$; log $C_2=0.114$.

				log C_2.
1.67	1.67	0.833	0.833	0.114
1.50	1.50	0.90	0.90	0.114
1.30	1.30	1.00	1.00	0.114
1 005	1.00	1.215	1.21	0.112
0.65	0.65	1.67	1.67	0.114
0.51	0.51	2.00	2.00	0.114
0.38	0.38	2.49	2.50	0.116
0.295	0.295	3.00	3.00	0.114
0.20	0.20	4.00	4.00	0.114
0.15	0.15	4.96	5.00	0.119
	Average, ...			0.1145

There is but one exception, in the chloroform-water-acetone series. As chloroform and water behave normally with alcohol (Table I.), water and acetone with benzol (Table VI.), the disturbing effect must be due to chloroform and acetone in presence of each other. I have not yet had time to investigate mixtures of chloroform and acetone in the absence of water, to determine whether they are abnormal in respect to any other physical properties. In the other five cases the agreement between observed

and calculated values is a remarkable one, well within the limits of experimental error, and this in spite of the wide range that the measurements cover. In the benzol-water-alcohol series the ratio of benzol to water varies as one to forty thousand; in the chloroform-water-alcohol series the ratio chloroform-water varies as one to twelve thousand. In the last measurement of Table I., the chloroform forms over 85 per cent by volume synthetically of the solution, so that in this instance we are well beyond the realms of the "dilute solutions," without noticing any disturbing effect due to "variations from the gas laws." The series benzol-water-alcohol is represented by a single curve; but it must not be thought that in this it forms a real exception to the other mixtures. Theoretically, there are two curves for this series; but the two happen to have the same direction, and therefore appear as one. The point where the precipitate ceases to be less dense than the original solution lies between the mixtures benzol 2.00 c.c., water 1.45 c.c., and benzol 3.57 c.c., water 1.00 c.c.

(*To be continued.*)

ON THE SECULAR MOTION OF A FREE MAGNETIC NEEDLE. II.[1]

BY L. A. BAUER.

ACCORDING to the principle explained in the first part of this article, all the curves on the two plates given have been constructed. In this paper no attempt at a detailed account of material employed, or of construction formulæ, can be given. It will suffice to present in tabular form the final construction data.

With the aid of Tables I. and II. appended, the curves on Plate I. were drawn. Only a part of the material collected by the writer has been utilized thus far. The object was to give only so much as was absolutely necessary to establish the conclusions here given. To be able to do this to the best advantage, it was the aim to select such stations as would exhibit most clearly the various phases of the secular variation. To further facilitate the study, most of the stations were selected approximately in latitude 40° N., and encircling the earth. The idea was to make a first attempt with the aid of such stations to follow a secular wave around the globe. It will be seen by turning to the plate, that the points (the intersections of the broken lines) which represent the mean values (D_0 and I_0) for each station have been placed in the latitude of that station. Owing to the difference in the size of the curves, the *real* difference in longitude between the various stations could not be preserved; the points (D_0, I_0) have, however, always been placed relative to each other in the proper longitude. This plate does not exhibit everything as clearly as the author would have wished; a larger scale could have been employed to good advantage. It should be remembered, however, that these curves have been drawn for the first time, and that in the nature

[1] A paper read before the National Academy of Sciences, Washington, April 16, 1895; concluded from page 465.

of the case this preliminary chart had to be more or less a trial one. The author hopes to be able to present, in the near future, a more complete and more comprehensive picture of the secular variation of geomagnetism.

But one more point with regard to the construction of the curves remains to be referred to. By turning to the Paris curve, it will be seen that the curve has been drawn from 1540 to 1890. For this station we possess declinations 1541–1895, but inclinations only from 1671 to 1895. How was it possible then to construct the curve from 1540 to 1671, since for this interval we possess the knowledge of but one ordinate, viz., declination? In this way. From the declination interpolation formula, it is found that the minimum value, $-9°.6$, or maximum easterly digression, was reached in about 1580. The secular variation curve must then have run in 1580 tangent to the declination ordinate corresponding to the value of $9°.6$ E. With the aid of this fact the curve can be extended backward from 1671 to 1580. Having done this, we can scale off an approximate value of the inclination for 1580, *i.e.* for a date for which we possess no such observations. This method of deriving inclinations for periods for which no data are at hand is here given for the first time. That the values thus derived can be relied upon to within 1° to 3°, is shown from the following. The scaled value of the inclination for Paris in 1580 lies between 70° and 72°; Norman observed at London in 1576, $71°.8$. This would imply a value for Paris lying between 70° and 71°. Hence the value $71°.0 \pm 1°.0$ cannot be far from the truth. With this value we can now extend the use of our interpolation formula, which had been established for the epoch 1671–1890. The formula gives for 1580, $71°.0$; we can consequently use it without great error back to 1580. The drawing of the curve can thus be undertaken without difficulty. Between the years 1541 and 1580 we know that the easterly declination was increasing; hence the secular variation curve, 1541–1580, must have approached the tangent point (1580) from the left. This part of the curve (1541–1580) is of course determined but very roughly, as no other values of the inclination were at hand than the extrapolated ones from the formula; but the *direction* of the motion

has been determined, and that is the chief object of these investigations. It is seen that from 1541 to 1890, *i.e.* for over 350 years, the curve proceeded clockwise. In precisely the same way, the London curve was extended from 1576 to 1540; the Rome curve from 1640 to 1508. In the latter case the direction has therefore been determined for well-nigh 400 years. For the epoch of maximum easterly declination, viz., 11°.6 E. in about 1570, we scale off an inclination of $62° \pm 1°.5$. As the earliest reliable inclination observed is that of Norman at London in 1576, $71°.8 \pm 1°$, the value scaled for Rome can boast of being one of the oldest that has come to our knowledge.

The first point to be observed is that for all the stations (24) on the chart, the motion proceeds throughout in the direction of the hands of a watch. This law has now been tested for more than one hundred observation series scattered over the earth. Where the data could be relied upon, or the series covered a sufficient time interval, the law was found to hold in every case. As the secular variation curves are not simple geometrical ones, but are broken up by a number of smaller waves, the writer does not mean to say that if the curves are drawn on a very large scale and, say, from month to month, the clockwise motion will obtain throughout all the very small loops of which the curve may be composed. It is simply asserted that in every case for the larger part of the curve, that is, for the secular variation broadly considered, the law holds for the interval for which the curves have been drawn. The law has been furthermore tested with the aid of the published magnetic charts, viz., the isogonic and the isoclinic charts for the epoch 1780–1885. From these charts the data were scaled for the intersections of the parallels of latitude 60° N., 40° N., 20° N., 0°, 20° S., 40° S., 60° S., with the meridians 20° apart. The secular variation curves for parallels of latitude 40° N., 0° and 40° S., *i.e.* $3 \times 18 = 54$ curves, have been drawn. For about a dozen there was some doubt as to the direction of the curve; in the rest, the clockwise motion again revealed itself. The doubtful curves were at such points on the earth's surface where the data are very meager, or where for the interval drawn the curve is in reality very small or nearly a straight line, *i.e.* the greater part of the

variation occurs in the inclination, the declination suffering very little change (see, for example, Manila curve). We believe then that we can safely draw our first conclusion, —

In consequence of the secular variation of geomagnetism, the north end of a freely suspended magnetic needle viewed from the center of suspension of the needle, moves on the whole earth in the direction of the hands of a watch.

So much care has been given the establishment of this law for two reasons: —

a. Because, knowing now that it is a law that the secular variation observes, the means are given herewith to judge somewhat as to the value of doubtful data.[1]

b. Because it is recognized that in the above law we have already one criterion, with which to decide between some of the causes of the secular variation that have been suggested. This law will doubtless play an important rôle in the next step of these investigations, — the mathematical examination and critical discussion of the possible causes.[2]

Very little can as yet be said as to the true geometric nature

[1] This can be elucidated by the following example. In 1885, Mr. Schott, the well-known geomagnetician, drew the secular variation curve for a mean station of New England, and for the interval 1820–1885. He knew approximately when the needle had reached its maximum easterly point, and also the value of the declination for this period. He could determine then the line to which the curve would have to run tangent, prior to 1820. Although he had had an experience of over forty years in terrestrial magnetic matters in the United States, he nevertheless did not dare to extend the curve back to this tangent line for the reason that he could not tell whether the extension would have its convex side turned downwards or upwards. Inclinations had been observed at Boston in 1780, but as they appeared doubtful to him no use was made of them. When the writer laid his preliminary investigations before the A. A. A. S. in 1892, he could then say how Mr. Schott's curve ought to be extended prior to 1820, viz., the convex side must be turned downwards in order to make the direction of the motion correspond to that obtaining at the European stations. Or, if the observed inclinations for 1780 were utilized, the curve would proceed in the direction as prescribed by the law. The curve for Boston was then exhibited for the period 1780–1885, and the direction prior to 1780 also indicated with the aid of the observed declinations. In the latter part of 1894, the writer's attention was called by Professor Cleveland Abbe to a work in which he obtained an observed inclination at Boston for the year 1722, this being probably the earliest observed inclination in the United States. With these data the Boston curve has now been laid down from 1722 to 1885. It is seen that the law of the motion obtains throughout, and that consequently the observed inclinations for 1780 cannot be far from the truth.

[2] See American Journal of Science for August, 1895, and following numbers.

of the secular curve. To the great question, however, whether it consists of a single branch or of several, an answer can be attempted. It may be remarked that it would be far more astonishing if the curve were a single closed one, than if it were shown to consist of branches or loops, since the daily and annual variation curves are known to be very complex indeed. Indications of loops are shown at two stations, Rome and the Azores. At the first station, by means of the so-called compass-charts of the 14th and 15th centuries, we are able to gain some knowledge of the magnetic bearing of the needle for that epoch. On these charts the directions from port to port are laid down magnetically. By comparison with charts on which astronomical directions are given, some knowledge of the magnetic declination for the period of the compass-charts can be obtained. Thus it was found that according to the compass-charts of 1436, by Andrea Bianco, the magnetic declination at Rome ought to be about 5° E. If we now assume that the law governing the secular variation of the declination from 1508 to 1890 holds good also from 1400 to 1508, we find that in 1400 the declination ought to be 12° W., and for 1436, 7° W. Prior to 1400 the westerly declination would be increasing. The material with which Bianco's charts were constructed is doubtless older than 1436. If we make the most favorable assumption, however, that the value 5° E. refers to 1436, we find a difference of 12° between compass-charts and formula. For 1400 the difference would be even 17°, and prior to that still greater. The best plausible explanation we can give of such large discrepancies, is that a different law of the secular variation came into play, or, in other words, a loop was described prior to 1500. A similar conclusion is reached at the Azores station, by considering the observed compass-bearing of Columbus in 1492, in the vicinity of these islands. Even if we assume that the value is erroneous so far as 6°, there still remains an outstanding difference between observation and formula of 8°. The writer does not mean to say that the foregoing considerations should be regarded as sufficient proofs, but simply as indications of loops. The singular points exhibited at some of the stations drawn, *e.g.* Acapulco and station, 40° N., 40° W., are doubtless due chiefly to inaccurate data. If

we compare the two curves 40° N., 60° W., and 40° N., 40° W., we find that they both observe the law of motion, and yet there is a striking difference between them. The former follows the United States type, the latter the European. Some remarkable change in the curve must have occurred between these two stations. The European type cannot change suddenly into that of the United States. The change must be a progressive one, and be made by means of loops or singular points. It is probable that the curves in this region will contain such singularities.

As to the period, nothing definite can be said as yet, and it moreover appears questionable whether there really exists a secular variation period, at the close of which the needle describes the same orbit it did before. The time interval between the epochs of maximum westerly and easterly digression of the needle can be determined for a number of stations to within about ten per cent. Thus this interval for London is, $1812-1580=232\pm10$ years; for Paris, $1809-1580=229\pm10$; for Rome, $1810-1570=240\pm15$. Accordingly, we might say that this interval for western Europe is about 235 years. For the eastern part of the United States the interval appears to be on an average about 150 years. We have a proof here at once that if the secular variation period has the same length all over the earth, we cannot then regard the interval between the extreme digressions as covering half the period; for, if this were so, then the period for Europe would be about 470 years, that for the eastern United States only about 300. *We are then forced to conclude that either the period is different for various portions of the earth, or that the secular curve is not a single closed curve, but consists of loops. This is our second conclusion.*

Now let us follow a secular wave around the earth. We will begin with London, and proceed in an easterly direction. The London curve seems to be approaching the upper extreme point, *i.e.* the inclination is diminishing with a speed that has become less and less ever since about 1810, or, in other words, it is nearing a minimum value which may be reached about the middle of the next century. Paris appears somewhat nearer this phase; Rome and Berlin still nearer, where it will probably take place shortly. At Tiflis this phase has already been passed, in about 1875. At

Bombay the minimum inclination phase has long ago occurred, and in about 1880 the curve had already reached its maximum easterly digression. Irkutsk, Peking, Manila, Petropawlowsk, station 40° N., 180° E., are either nearing the maximum inclination phase or have already passed through it. It is extremely difficult to say through what phase the San Francisco curve has passed, for the reason that in the Pacific Ocean and on the western coast of the United States, secular waves of opposite phase, but of different amplitudes, seem to make their appearance, and consequently partially annihilate each other. St. Louis, Boston, Bermudas, 40° N., 60° W., have passed through their extreme lower points, and are approaching now the maximum westerly digression. The latter will take place in time in the reversed order of the stations. At station 40° N., 40° W., the maximum westerly elongation has just occurred, or will occur soon. At the Azores it took place about 1852, and the needle, as at London, is bent now upon reaching its upper extreme position. We have now followed a wave around the earth.

Our next conclusion is that the secular variation curves appear to develop themselves more and more as we go around the earth eastwardly; or, in other words, the secular wave appears to travel in the main, roughly speaking, westward. We might then conclude that by obtaining a composite of the various parts of the successive secular variation curves for stations, somewhere near a parallel of latitude, we could get the total curve. If we do this mentally, we shall find at once that the curve constructed thus is not a single looped one, as there are two regions where the needle is passing through a maximum inclination phase, and likewise two where the maximum easterly (or minimum westerly) phase is taking place.

But what does this unfolding of the secular variation curve, with eastward progression, imply? If this continuous development has been caused by a wave traveling from east to west around the earth, would it not follow that if we made an instantaneous circuit of the earth in an easterly direction with a free magnetic needle, the same phenomenon would unfold itself from station to station, as occurs at any fixed station in the lapse of time? Let us see if

this be true. In 1885 we will start from a point, the latitude of which is 40° N., and longitude 0°. We will take with us a free magnetic needle, and travel with it eastwardly around the earth along the parallel, 40° N. At points distant 20° in longitude, we will observe the direction of the needle; measuring the declination and the inclination. Throughout the circuit the needle is constantly changing its direction. If we now suppose that the center of suspension of the needle is fixed, but the needle itself subject to the changes encountered during the circuit, we can construct the curve described by the north end, in a manner analogous to that which was followed in drawing the secular variation curves. We thus obtain the striking curve exhibited on Plate II., — the heavy curve on the left. The only difference from the secular variation curves is, that instead of time we now have longitude marking the various points of the curve. Instead of actually making the circuit, we can obtain the necessary data from the excellent isogonic and isoclinic charts for 1885, constructed by Professor Neumayer, director of the German Naval Observatory. The broken places correspond to the data for the points 20° distant in longitude, and lying in latitude 40° N. It will be observed that this curve also proceeds throughout in the direction of the hands of a watch; even the small loop follows this law. Furthermore, the part without the loop exhibits a great similarity to the secular variation curves for London, Paris, and Rome. Is this mere accident? Suppose we make our circuit along other latitudes, or for other epochs, — will a similar condition of things obtain? To answer this the writer has drawn first the instantaneous curves for latitudes 75° N., 70° N., 60° N., 50° N., 40° N., 20° N., 0°, 20° S., 40° S., 60° S., from data of 1885, and secondly the curves for 40° N., 0°, and 40° S., for the additional years 1780 and 1829. In every case the direction of motion was clockwise. On Plate II., these latter curves are drawn for the three years; the necessary data are given in Table III. It will be observed that although the nine curves present many irregularities and singularities, they nevertheless all unite to exhibit a hitherto unsuspected law. Before we formulate the new conclusion to be drawn, let us see what we shall get if we make the circuit in some other way than along a

parallel of latitude. Of course the circuit must always be made eastwardly as the waves come, generally speaking, from the east. Likewise is it apparent that for every circuit we shall get a closed curve. At first sight it might appear more natural to make the circuit along a magnetic parallel. If we define the latter as an isoclinic, then the curve will reduce to a straight line, since along an isoclinic the inclination is constant, the declination alone varying. If we regard an equipotential line as a magnetic parallel, and make the circuit *e.g.* along the zero equipotential line or along the magnetic equator, we get the peculiar curve given on Plate II., middle figure. This curve has a large loop proceeding clockwise, and two small loops, one going anti-clockwise and the other clockwise. It will at once be seen that this curve does not present such a striking similarity to the secular variation curve. It seems then that the circuit must be made somewhere near a parallel of latitude. Is this not an indication that the secular variation is in some manner connected with the rotation of our mighty geomagnet? This matter is at present being investigated. Our next conclusion is then: —

The north end of a free magnetic needle, viewed from the center of suspension of the needle, moves clockwise in making an instantaneous easterly circuit of the earth along a parallel of latitude; or,

The north end of a free magnetic needle, whose center of suspension is fixed in space close to the earth's surface, will describe a curve as the earth rotates under it, which as viewed from the center of suspension of the needle, moves anti-clockwise.[1]

In the foregoing we have already hinted at a connection between the secular variation and the distribution of geomagnetism, in that they both observe similar laws. This connection is also revealed in another way. By turning to Plate I., it will be seen that the largest secular curves seem to occur at stations somewhere near the equator, — see Ascension and St. Helena islands. Likewise the curve for Rome is smaller than the one for Cape Town, lying in

[1] The original formulation of this law was given in the first form. The curves are indicated then on Plate II., as proceeding clockwise. For the second form the arrows would have to be reversed. The motion is now reversed since the earth rotating from west to east is equivalent to making a *westerly* circuit of the earth with the needle.

about the same latitude south. Turning now to Plate II., it will be seen that a precisely similar condition of things prevails. Thus the largest instantaneous curve is described at the equator; likewise the curves for 40° S. are larger than those for 40° N. A connection seems to prevail also with regard to the loop in the northern hemisphere. This cannot be exhibited here, however.

Our final conclusion is then:—

The secular variation and the prevailing distribution of geomagnetism appear to be closely related.

TABLE I.

GENERAL DATA.

Station.	Latitude.	Longitude from Greenwich.	Year of first observation.		Maximum D		Minimum D		Maximum I		Minimum I	
			D	I	Date.[1]	Amount.	Date.[1]	Amount.	Date.[1]	Amount.	Date.[1]	Amount.
	° ′	° ′				°		°		°		
London	51 30 N.	0 07 W.	1580	1576	1812	+24.2	1580	−11.0	1688	74.4[3]	—	—
Paris	48 50	2 20 E.	1541	1671	1809	+22.6	1580	− 9.6	1680	74.7	—	—
Geneva	46 12	6 09	1797	1775	—	—	—	—	—	—	—	—
Basle	47 34	7 36	1691	1758	1810	+21.8	—	—	—	—	—	—
Rome	41 53	12 26	1508	1640	1810	+17.1	1570	−11.6	1690	66.9	—	—
Berlin	52 32	13 23	1731	1769	1802	+18.1	—	—	—	—	—	—
Tiflis	41 43	44 50	1833	1833	—	—	—	—	—	—	1875	55.4
Bombay	18 54	72 49	1676	1845	—	—	1880	− 1.0	—	—	—	—
Irkutsk	52 17	104 20	1820	1805	—	—	1875	− 2.8	—	—	—	—
Peking	39 57	116 28	1755	1831	—	—	—	—	—	—	—	—
Manila	14 35	120 58	1829	1767	—	—	—	—	—	—	—	—
Petropawlowsk	53 01	158 43	1779	1779	—	—	1771	− 6.3	1890(?)	64 3(?)	—	—
Pacific Ocean	40 00	180 00	1780	1780	—	—	—	—	—	—	—	—
San Francisco	37 48	122 27 W.	1783	1815	—	—	1893(?)	−16.6	—	—	—	—
Acapulco	16 50	99 52	1744	1791	—	—	1846	− 8.9	—	—	—	—
St. Louis	38 38	90 12	1835	1819	—	—	1822	− 8.9	—	—	—	—
Boston[2]	42 22	71 06	1700	1722	1919(?)	+12.3(?)	1780	+ 6.7	1850	74.4	—	—
Bermuda Is.	32 23	64 43	1831	1831	—	—	—	—	—	—	—	—
Atlantic Ocean	40 00	60 00	1780	1780	—	—	—	—	—	—	—	—
Atlantic Ocean	40 00	40 00	1780	1780	—	—	—	—	—	—	—	—
Azores Is.	38 32	28 39	1589	1775	1852	+27.4	1612(?)	+ 2.5(?)	1700(?)	72.6(?)	—	—
Ascension Is.	7 56 S.	14 23 W.	1678	1754	1885	+22.9	—	—	—	—	—	—
St. Helena Is.	15 55	5 44	1610	1754	1890(?)	+24.2(?)	1599(?)	− 6.4(?)	—	—	—	—
Rio de Janeiro	22 55	43 10	1751	1751	—	—	—	—	1882(?)	−11.8(?)	—	—
Cape Town	33 56	18 29 E.	1605	1751	1870	+29.6	1575(?)	− 0.3(?)	—	—	—	—
Melbourne	37 50	144 59	1858	1858	—	—	—	—	—	—	—	—

[1] Approximate. [2] And Cambridge. [3] Probably about 1° too low.

TABLE II.

CONSTRUCTION DATA OF THE SECULAR VARIATION CURVES. (PLATE I.)

London.			Paris.			Rome.			Boston.		
Date.	D[1]	I[1]	Date.	D	I[1]	Date.	D	I[1]	Date.	D	I[3]
	°	°		°	°		°	°		°	°
						1508	− 6.53	+58.8(?)			
						20	− 8.23	+59.4(?)			
1540	− 7.2(?)	+69.4(?)	1540	− 8.2	+68.4(?)	40	−10.47	+60.5(?)			
60	− 9.6(?)	+70.4(?)	60	− 9.3	+69.7(?)	60	−11.61	+61.7(?)			
80	−10.93	+71.36	80	− 9.6	+71.0(?)	80	−11.41	+62.9(?)			
1600	−10.13	+72.27	1600	− 8.8	+72.2(?)	1600	− 9.88	+64.1(?)			
20	− 7.24	+73.06	20	− 6.9	+73.2(?)	20	− 7.29	+65.1(?)			
40	− 3.27	+73.70	40	− 4.42	+74.0(?)	40	− 3.86	+66.00			
60	+ 0.59	+74.14	60	− 0.86	+74.6(?)	60	+ 0.01	+66.59			
80	+ 3.89	+74.36	80	+ 3.47	+74.73	80	+ 4.01	+66.89			
1700	+ 7.08	+74.34	1700	+ 7.99	+74.56	1700	+ 7.77	+66.87	1700	+10.0	—
20	+10.97	+74.09	20	+12.27	+74.06	20	+11.02	+66.52	20	+ 8.7	—
40	+15.30	+73.62	40	+15.83	+73.27	40	+13 63	+65.88	40	+ 7.71	—
60	+19.57	+72.96	60	+18.76	+72.23	60	+15.51	+64.99	60	+ 6.98	—
80	+22.65	+72.14	80	+20.87	+71.03	80	+16.64	+63.91	80	+ 6.68	+69.7
1800	+24.07	+71.23	1800	+22.12	+69.75	1800	+17.06	+62.71	1800	+ 7.00	+70.9
20	+24.09	+70.26	20	+22.40	+68.47	20	+16.77	+61.49	20	+ 7.88	+72.9
40	+23.22	+69.30	40	+21.38	+67.28	40	+15.84	+60.31	40	+ 9.15	+74.25
60	+21.55	+68.41	60	+19.54	+66.29	60	+14.23	+59 26	60	+10.48	+74.42
80	+18.73	+67.63	80	+16.76	+65.55	80	+11.77	+58.43	80	+11.53	+73.65
90	+17.57[2]	+67.21	90	+15.16	+65.27	90	+10.57	+58.10	85	+11.7	+73.12

[1] Values marked questionable have been derived by extrapolation from the secular variation formulæ.

[2] This value has not been derived from the formula but from the observations.

[3] Inclination in 1722.7 was 68°.37 ± 1°.

TABLE II. — (*Continued.*)

Station.	1750		1760		1780		1800		1820	
	D	*I*	*D*	*I*	*D*	*I*	*D*	*I*	*D*	*I*
	°	°	°	°	°	°	°	°	°	°
Fayal Is. (Azores)	—	—	—	—	+21.25	+70.26	+24.08	+69.10	+26.13	+67.83
Berlin	+14.98	—	+16.07	—	+17.56	+71.92	+18.12	+70.33	+17.74	+69.02
Geneva[1]	—	—	—	—	—	+69.07	+21.50	+67.47	+20.59	+66.07
Basle[1]	—	—	+18.	+70.35	+20.	+68.90	+21.7	+67.61	+21.7	+66.47
Peking	—	—	—	—	—	—	—	—	—	—
Manila	—	—	—	—	—	—	− 0.18	+12.9	− 0.38	+15.0
Petropawlowsk .	—	—	—	—	− 6.3	+63.0	− 5.7	+63.4	− 4.7	+63.8
San Francisco . .	—	—	—	—	−12.6	—	−13.6	—	−14.54	—
St. Louis . . .	—	—	—	—	—	—	—	—	—	+69.86
Ascension Is. . .	+ 6.7	+11.3	+ 8.4	+10.3	+11.6	+ 8.2	+14.0	+ 5.6	+16.4	+ 3.0
St. Helena Is. . .	+ 9 77	− 9.9	+11.70	−10.5	+14.59	+12.0	+17.51	−13.5	+20.01	−15.4
Rio de Janeiro .	− 7.27	−20.41	− 7.44	−19.50	− 6.84	−17.98	− 5.42	−16.50	− 3.46	−14.66
Cape Town . . .	+19.10	−43.6	+20.5	−44.2	+23.2	−46.53	+25.4	−48.85	+27.2	−51.01
Melbourne . . .	—	—	—	—	—	—	—	—	—	—

Station.	1840		1860		1880		1890			
	D	*I*	*D*	*I*	*D*	*I*	*D*	*I*		
	°	°	°	°	°	°	°	°		
Fayal Is. (Azores)	+27.22	+66.57	+27.33	+65.39	+26.40	+64.38	+25.58	+63.96		
Berlin	+16.43	+68.00	+14.18	+67.25	+11.0	+66.8	—	—		
Geneva[1]	+19.21	+64.89	+17.34	+63.91	+15.00	+63.13	+13.65	+62.81		
Basle[1]	+20.2	+65.49	+18.	+64.66	+15.2	+63.99	+13.0	+63.47		
Peking	+ 2.06	+55.5	+ 2.32	+56.6	+ 2.68	+56.8 ?	+ 2.9	+56.5 ?		
Manila	− 0.54	+16.5	− 0.70	+17.4	− 0.78	+17.7	− 0 80	+17.6		
Petropawlowsk .	− 3.4	+64.1	− 2.0	+64.2	− 0.4	+64.3	+ 0.5	+64.3		
San Francisco . .	−15.42	+62.4	−16.10	+62.4	−16.51	+62.3	−16.58	+62.3		
St. Louis . . .	− 8.6	+69.46	− 7.7	+69.66	− 6.4	+69.26	− 5.6	—		
Ascension Is. . .	+18.8	− 0.4	+21.4	− 4.7	+22.9	− 9.3	+23.0	−11.6		
St. Helena Is. . .	+22.00	−18.0	+23.41	−22.0	+24.11	−26.5	+24.21	−29.65		
Rio de Janeiro .	− 1.01	−12.89	+ 1.71	−12.01	+ 4.52	−12.11	+ 5.88	−12.29		
Cape Town . . .	+28.8	−53.03	+29.7	−54.89	+29.6	−56.59	+29.2	−57.39		
Melbourne . . .	—	—	− 8.50	−67.09	− 8.14	−67.08	− 7.85	−67.03		

[1] These curves could not be given on Plate I. for want of space.

TABLE II. — (*Concluded.*)

Acapulco.			Bermudas.			40° N., 60° W. of Gr.			40° N., 40° W. of Gr.		
Date.	*D*	*I*	Date.	*D*	*I*	Date.	*D*	*I*	Date.	*D*	*I*
	°	°		°	°		°	°		°	°
1791.3	−7.7	+36.1	1837.5	+6.93	+65.82	1780.0	+ 8.2	+68.8	1700	+ 5.8	+73.0
1803.2	−7.8	+38.9	1873.4	+7.17	+66.43	1830.0	+12.7	+72.3	1780	+14.5	+70.0
1838.1	−8.2	+38.0	1890.4	+8.07	+64.79	1885.0	+18.0	+71.3	1830	+22.7	+70.1
1866.4	−8.4	+39.9							1842	+24.5	+70.9
1880.9	−7.9	+40.1							1885	+25.4	+68.1
Tiflis.			**Bombay.**			**Irkutsk.**			**40° N., 180° E. of Gr.**		
Date.	*D*	*I*	Date.	*D*	*I*	Date.	*D*	*I*	Date.	*D*	*I*
	°	°	°	°	°		°	°		°	°
1840.0	+2.55	+55.90	1846.0	0.00	+18.14	1828.5	−1.74	+68.04	1780	−12.5	+49.5
1871.0	−0.20	+55.50	1856.0	−0.32	+19.10	1870.0	−2.70	+69.80	1830	−12.4	+54.3
1875.0	−0.55	+55.45	1870.0	−0.79	+19.50	1888.0	−2.28	+70.08	1842	−12.6	+55.0
1882.0	−0.90	+55.51	1880.0	−0.96	+19.70				1885	−11.5	+53.8
1888.0	−1.23	+55.66	1890.0	−0.81	+19.80						

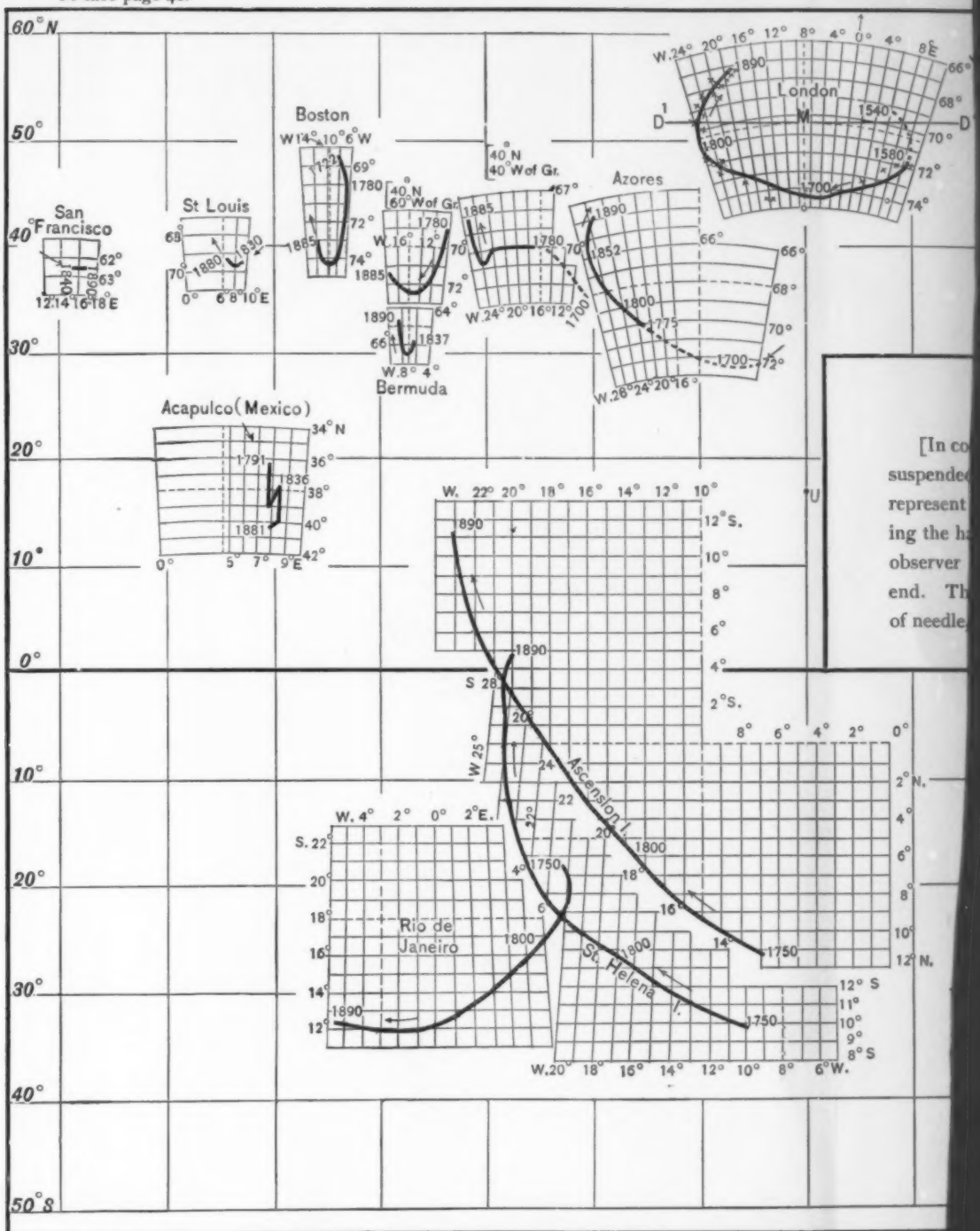
TABLE III.

CONSTRUCTION DATA[1] OF THE INSTANTANEOUS CURVES (PLATE II.).

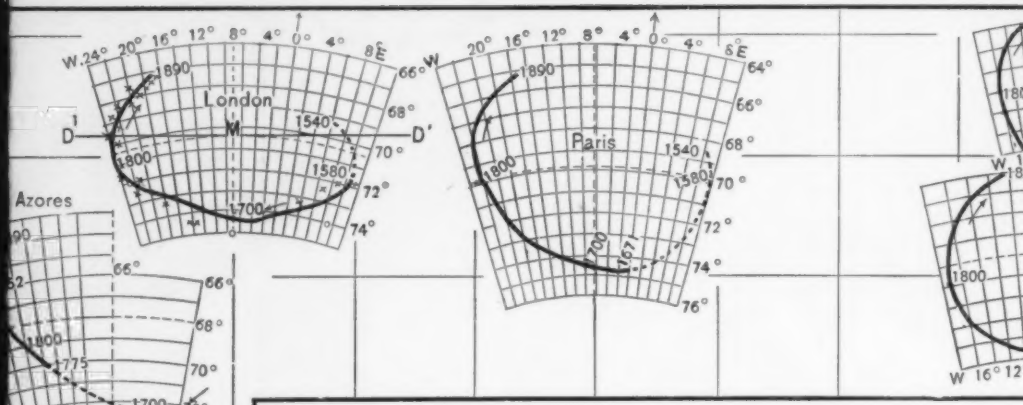
Long. E. of Gr.	Latitude 40° N.						Equator.						40° S.						Zero equipotential line.	
	1780		1829		1885		1780		1829		1885		1780		1829		1885		1885	
	D	*I*	*D*	*I*	*D*	*I*	*D*	*I*	*D*	*I*	*D*	*I*	*D*	*I*	*D*	*I*	*D*	*I*	*D*	*I*
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	+19.8	+66.2	+22.0	+59.6	+15.6	+58.2	+16.5	+15.5	+21.3	+ 6.5	+19.0	− 8.2	+11.3	−44.3	+20.3	−47.7	+27.4	−51.0	+19.5	− 9.5
20	+15.3	+60.3	+13.6	+54.0	+ 8.0	+54.7	+18.7	+ 0.3	+20.1	−12.5	+13.3	−21.8	+22.6	−51.0	+28.5	−56.1	+31.7	−58.8	+12.0	−11.0
40	+ 8.7	+54.6	+ 4.0	+51.0	+ 0.7	+53.0	+14.5	−13.0	+12.4	−24.7	+ 7.6	−23.8	+29.2	−60.0	+30.8	−62.9	+31.2	−64.0	+ 6.0	− 7.0
60	+ 3.4	+51.2	− 3.4	+51.9	− 4.6	+54.1	+ 7.0	−23.0	+ 3.6	−26.5	+ 3.	−22.0	+27.0	−69.?	+27.7	−66.9	+27.0	−67.0	+ 1.7	− 3.0
80	+ 1.1	+51.2	− 5.6	+55.1	− 6.5	+55.3	− 0.0	−20.7	− 0.7	−21.9	0.0	−20.4	+20.5	−69.?	+22.0	−68.7	+23.0	−68.5	− 1.1	− 1.0
100	+ 2.7	+53.8	− 2.4	+57.3	− 3.0	+56.6	− 0.2	−16.3	− 0.8	−17.8	− 2.2	−17.3	+16.8	−67.?	+15.0	−69.6	+17.0	−69.2	− 3.0	+ 1.0
120	+ 0.7	+54.4	+ 1.9	+56.8	+ 3.6	+56.8	− 0.2	−16.0	− 0.8	−17.1	− 2.1	−12.8	+10.8	−67.5?	+ 5.0	−69.9	+ 5.2	−70.0	− 1.8	+ 4.0
140	− 1.8	+51.8	+ 2.0	+54.5	+ 5.3	+54.0	− 3.5	−14.5	− 4.1	−16.9	− 3.2	−12.4	− 1.3	−67.5?	− 6.6	−68.2	− 5.8	−69.5	− 2.0	+ 5.0
160	− 6.6	+48.6	− 2.9	+53.1	− 2.5	+51.8	− 9.1	−11.7	− 8.9	−12.5	− 7.3	−10.0	−12.1	−66.2	−13.4	−64.1	−12.9	−66.8	− 6.8	+ 3.0
180	−12.5	+49.5	−10.0	+54.3	−11.5	+53.8	−10.5	− 7.6	−10.8	− 3.7	− 8.7	− 2.8	−14.0	−62.7	−13.5	−59.5	−14.5	−63.2	− 8.8	+ 4.5
200	−15.6	+52.5	−15.5	+57.7	−14.8	+57.3	− 7.1	− 5.0	− 8.1	+ 3.1	− 6.6	+ 1.5	−10.9	−60.3	− 9.9	−57.1	−12.7	−60.0	− 6.7	+ 1.2
220	−15.9	+57.5	−17.6	+61.8	−17.1	+60.8	− 3.9	− 4.5	− 5.3	+ 4.3	− 3.0	+ 2.4	− 6.6	−60.5	− 7.8	−57.4	−12.2	−58.2	− 2.5	− 1.5
240	−13.3	+63.0	−16.2	+65.6	−17.6	+64.8	− 1.7	− 2.4	− 6.1	+ 4.3	− 5.2	+ 4.2	− 2.3	−62.2	−10.5	−58.1	−13.7	−56.0	− 5.6	− 4.0
260	− 7.3	+67.0	−11.2	+69.1	−12.4	+69.0	− 5.3	+ 3.3	− 8.6	+ 8.5	− 7.6	+ 8.2	− 6.8	−61.6	−15.9	−56.0	−18.0	−52.2	− 9.2	− 5.0
280	− 0.7	+68.7	− 1.5	+72.5	+ 1.8	+71.4	− 7.6	+11.5	− 8.9	+17.1	− 7.4	+15.3	−14.0	−56.0	−17.8	−50.6	−19.4	−44.5	−10.5	− 5.0
300	+ 8.2	+68.8	+11.7	+72.3	+18.0	+71.3	− 6.3	+20.0	− 4.2	+25.7	− 1.8	+23.8	−17.5	−50.5	−13.5	−44.0	−11.1	−35.4	− 4.7	− 1.0
320	+14.5	+70.0	+22.4	+70.1	+25.4	+68.1	− 0.6	+24.4	+ 5.2	+28.7	+ 9.6	+25.8	−10.5	−45.5	− 4.0	−39.8	+ 3.1	−33.8	+ 9.0	+ 3.5
340	+19.7	+69.2	+25.8	+65.6	+23.6	+63.2	+ 8.8	+24.8	+15.5	+22.4	+21.0	+12.7	+ 0.3	−42.7	+ 8.4	−41.2	+17.4	−41.5	+21.5	− 1.0
Mean	+ 1.1	+58.8	+ 1.0	+60.1	+ 0.7	+59.7	+ 0.5	− 1.9	+ 0.6	− 1.8	+ 1.0	− 3.2	+ 2.3	−59.1	+ 2.5	−57.7	+ 3.5	−57.2	+ 0.4	− 1.5

[1] These data were obtained by scaling published isogonic and isoclinic charts. The figures as given are on the average liable to an uncertainty of at least $\frac{1}{2}°-1°$. Those for 1780 were derived from Hansteen's isogonic charts of 1770 and 1787, and his isoclinic chart for 1780 as given in the Atlas to his "Magnetismus der Erde," Christiania, 1819. The mean values of the declinations obtained from the first two charts when giving double weight to the 1787 scalings, were regarded as corresponding to the epoch of the inclinations scaled from the chart 1780. The 1829 data were extracted from Erman and Petersen's "Grundlagen der Gaussischen Theorie und die Erscheinungen des Erdmagnetismus im Jahre 1829," Berlin, 1874. They are the computed results as based upon Gauss's potential theory of terrestrial magnetism. The 1885 figures were derived by scaling the excellent charts of Neumayer as given in his "Atlas des Erdmagnetismus," Gotha, 1891. Some interesting results follow from the above table, *e.g.* that the mean declination for a parallel of latitude is always plus or westerly. The discussion of these results will be given in a future paper in connection with the data from 27 charts.

To face page 48.

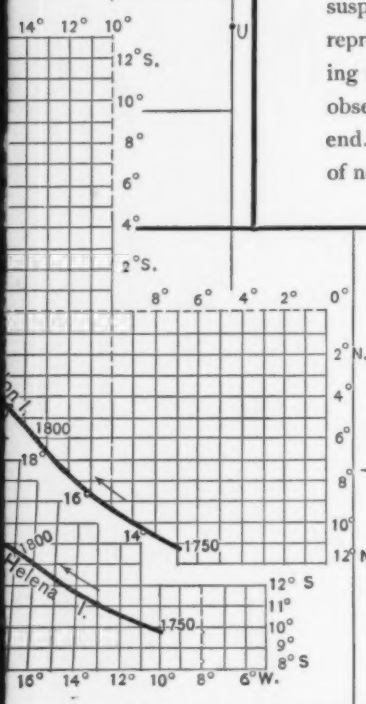


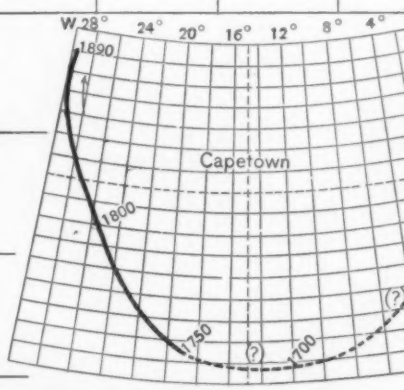
L. A. BAUER

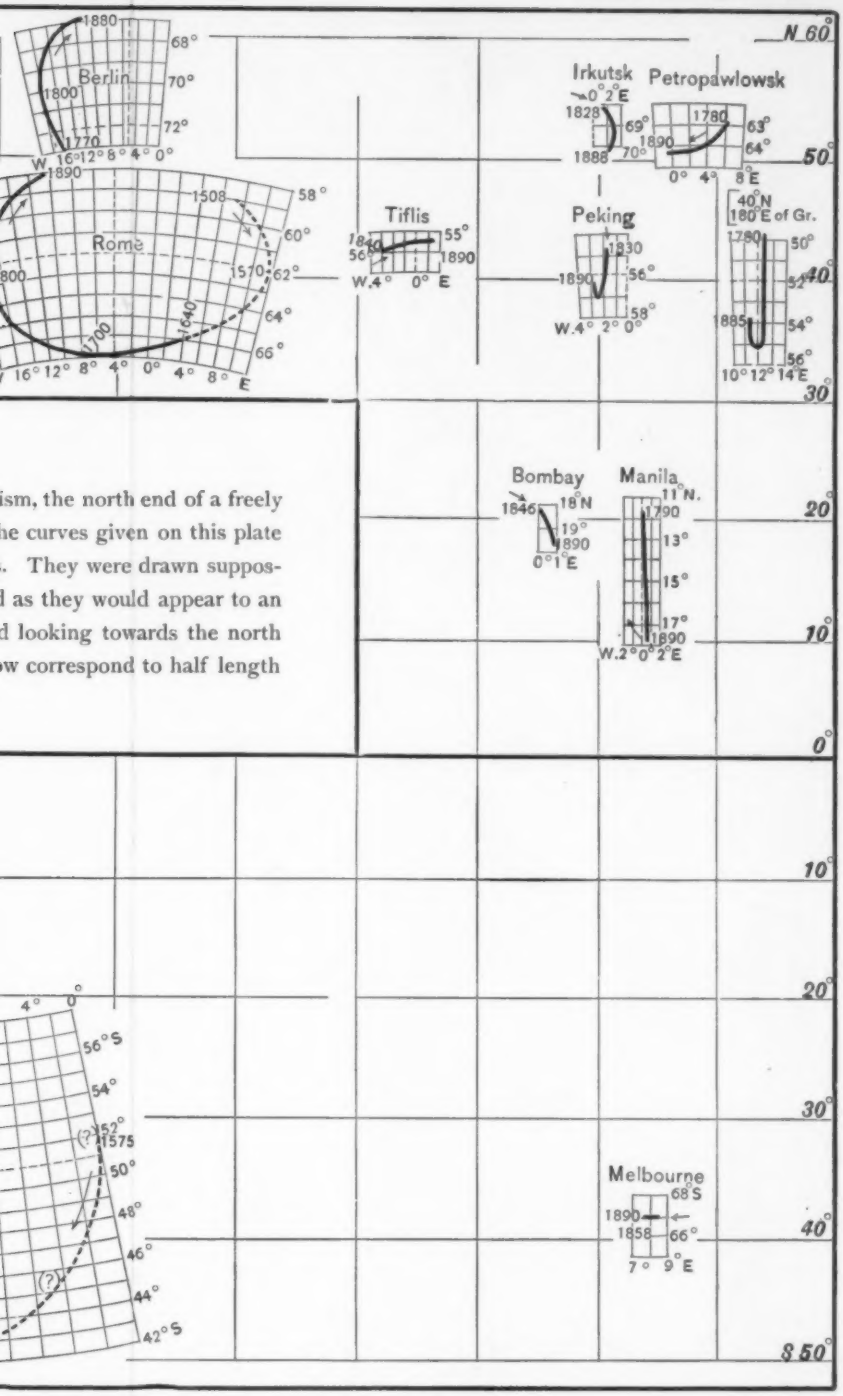


SECULAR CURVES.

[In consequence of the secular variation of terrestrial magnetism, th
suspended magnetic needle describes some kind of a curve. The curv
represent the central projections of these secular variation curves. The
ing the half-length of the needle to be 20 cm. (7.88 inches), and as th
observer standing at the point of suspension of the needle and look
end. The original scale has been reduced so that the curves now cor
of needle, 15.5 cm. or 6.1 inches.]







ism, the north end of a freely
he curves given on this plate
. They were drawn suppos-
d as they would appear to an
d looking towards the north
w correspond to half length

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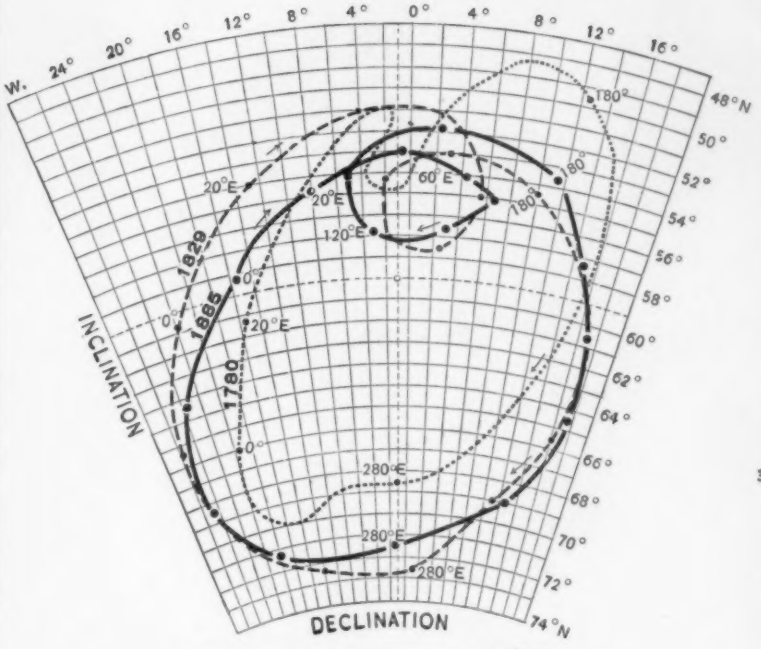
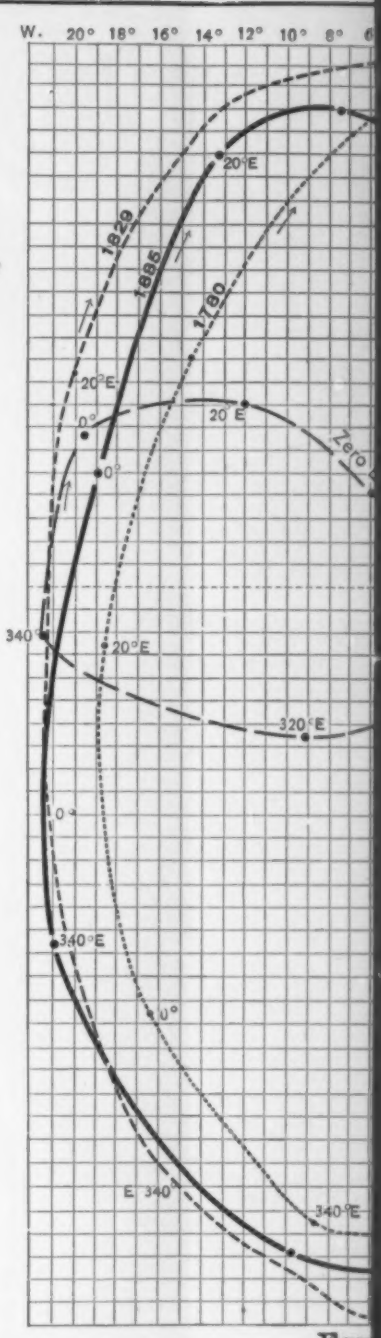
INSTANTANEOUS



Parallel of Latitude 40° N.

[In consequence of the earth's rotation, the north end of a free magnetic needle whose center of suspension is fixed in space close to the earth's surface describes some kind of a curve. The curves given on this plate represent the central projections of these curves. They are drawn supposing the half-length of the needle to be 20 cm. (7.88 inches), and as they would appear to an observer standing at the point of suspension of the needle and looking towards the north end.]



Equ

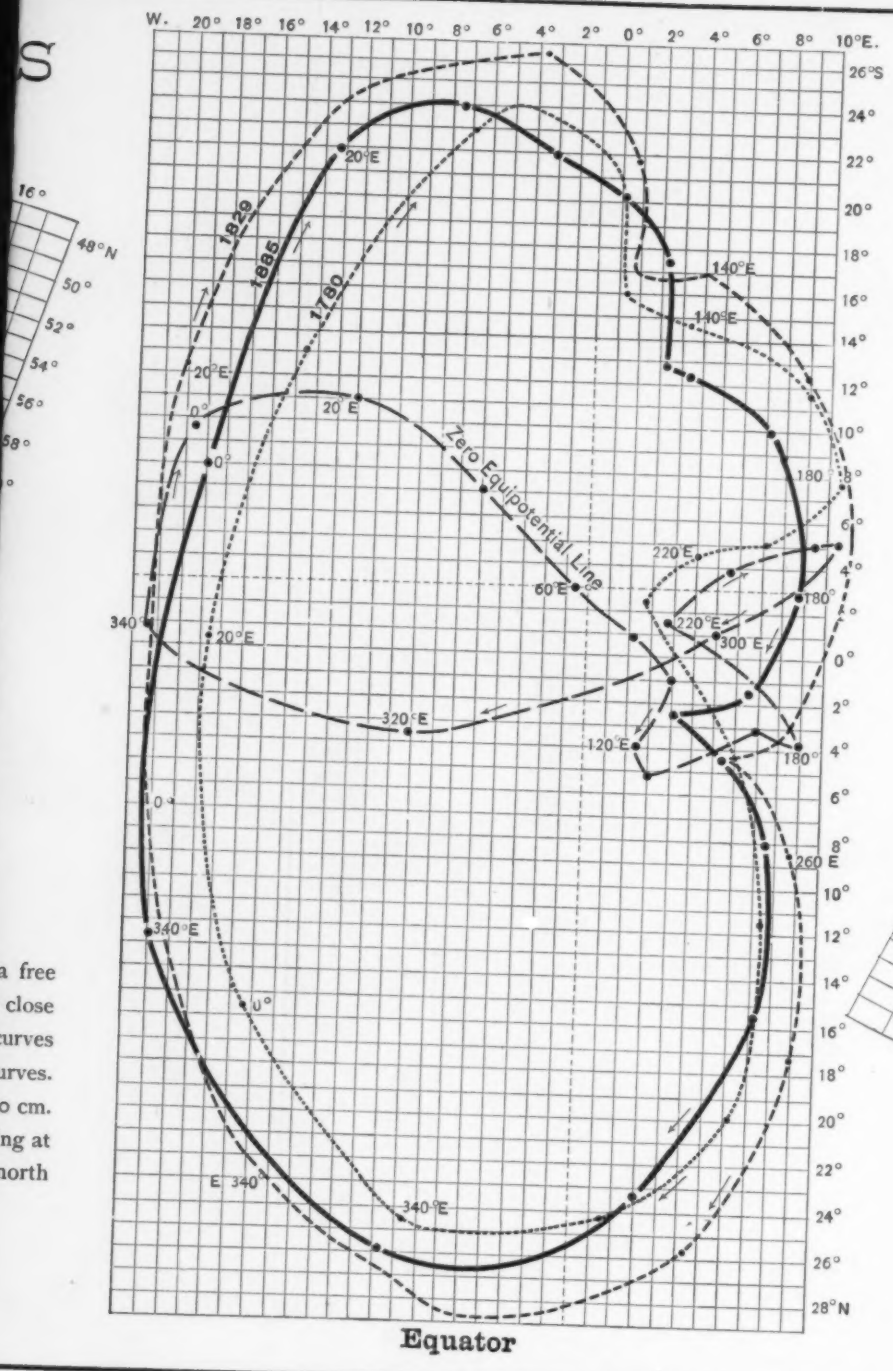
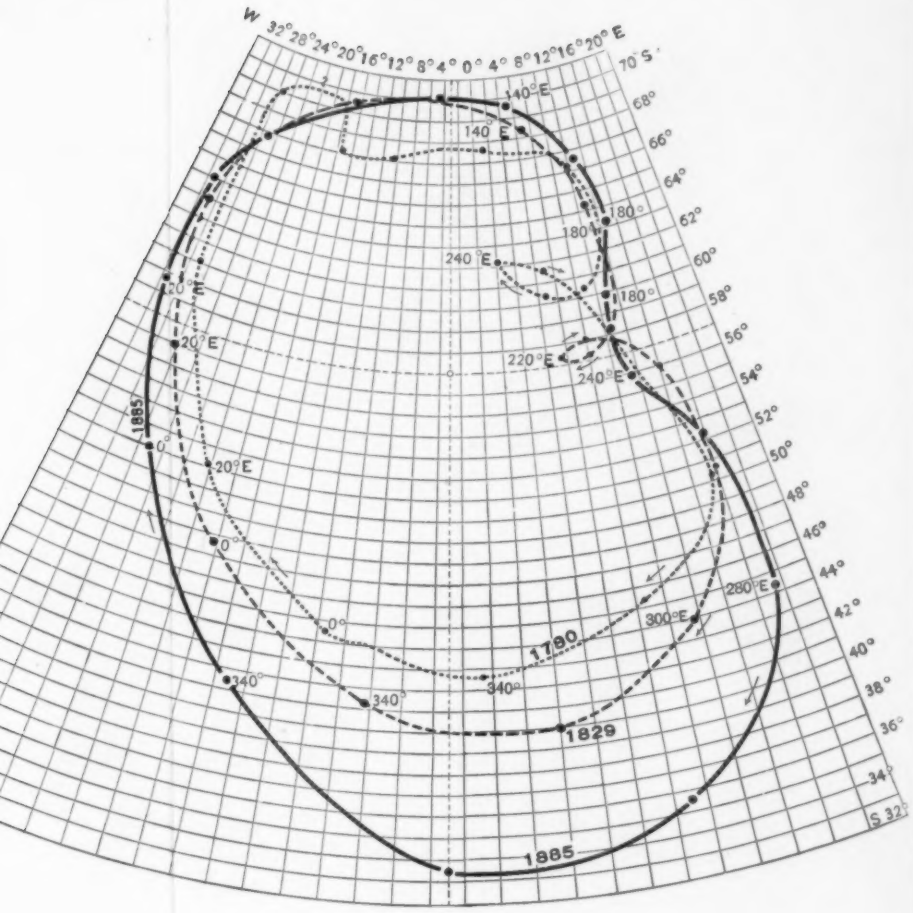

W. 20° 18° 16° 14° 12° 10° 8° 6° 4° 2° 0° 2° 4° 6° 8° 10°E.
26°S
24°
22°
20°
18°
16°
14°
12°
10°
8°
6°
4°
2°
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
20°
22°
24°
26°
28°N
1829
1885
1780
20°E
20°E
20°E
0°
0°
20°E
Zero Equipotential Line
60°E
340°
20°E
320°E
340°E
0°
340°E
0°
340°E
140°E
140°E
180°
220°E
180°
220°E
300°E
120°E
180°
260°E
Equator

Plate 2.

CURVES



Parallel of Latitude 40°S.

A GALVANOMETER FOR PHOTOGRAPHING ALTERNATING CURRENT CURVES.

By H. J. Hotchkiss and F. E. Millis.

IN the latter part of 1893 one of the writers undertook an experimental study of the extra currents corresponding to the exponential term in the common expression for the current curve produced upon closing a circuit containing an harmonic electromotive force, resistance, and self-induction; and the other, an investigation of the variations of current and armature lag in synchronous motors when the load is suddenly changed. For the experimental study of these subjects we wished an instrument which would readily and accurately record the alternating current curve from the instant of changing the conditions until the permanent form was reached. Photographing on a moving plate the spot of light reflected from a modified form of M. Blondel's oscillograph[1] seemed to lend itself best to our purpose. The modified form is essentially a galvanometer having a very small needle of soft iron, a very strong field for the directing force, and a single small coil carrying the alternating current which deflects the needle.

That the curve thus obtained might not be distorted by the inertia of the vibrating needle and mirror, both were as small as they could be conveniently made, and were mounted with their greatest dimensions in the axis of vibration. That the natural period of the needle may not affect the shape of the curve, it should be *much* less than that of the current.

We experimented with many forms of mounting for the needle, and finally succeeded in getting one sufficiently sensitive which made a little more than 3500 double vibrations per second. The frequency of the current experimented upon in the study of extra

[1] Electrician, Lond., Vol. XXXI., March 17, 1893; page 571.

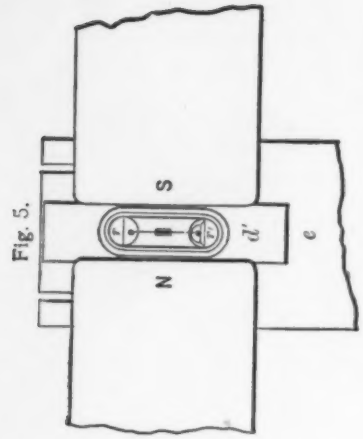


Fig. 5.

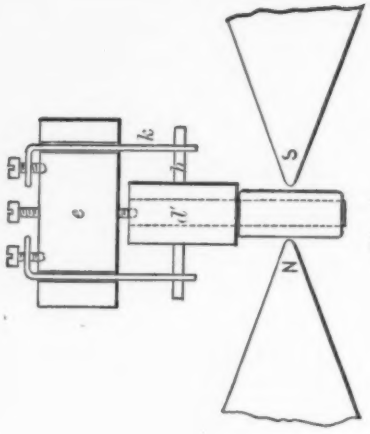


Fig. 4.

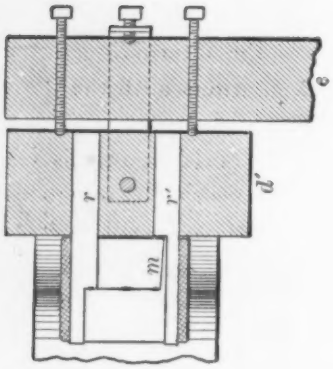


Fig. 6.

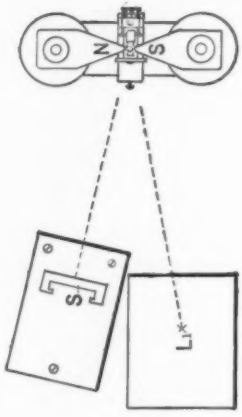


Fig. 3.

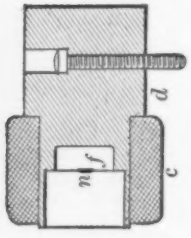


Fig. 2.

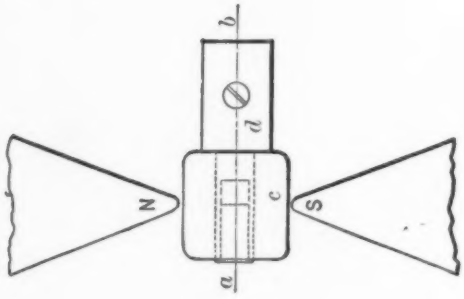


Fig. 1.

currents was 120 complete alternations per second, and it was not possible to detect the slightest distortion of the curve due to the combination of the period of the needle with that of the current.

Figures 1 and 2 show the manner of mounting the needle and its position between the poles *N*, *S*, of the electromagnet. Figure 1 is a horizontal projection, and Fig. 2 a vertical section through Fig. 1 along the line *ab*.[1] The needle *n*, and the small coil *c* around it, were supported upon a vulcanite block *d*, having a rectangular hole 6 × 15 mm. cut into one end. The latter was cut down until the walls were quite thin, in order to bring the coil close to the needle and not to separate the pole pieces too far. Within, shoulders were left, on which to mount the coarse quartz fiber *f*, to the middle of which the needle and mirror were fastened, on opposite sides, with shellac. The needle was made from a piece of soft iron such as is used for armature cores. The mirror was cut from a microscope cover-glass silvered. The dimensions of the parts are given in column I. of the table.

The small coil *c*, around the needle, contained about 480 turns of copper wire heavy enough to carry 3 or 4 amperes. With this apparatus an alternating current of 2.35 amperes gave an amplitude of about 18 mm. on the photographic plate.

A top view of the electromagnet and photographing apparatus, drawn to a scale of about one-fifteenth, is given in Fig. 3. The wrought iron cores, about 2 inches in diameter, were connected at the bottom by a large yoke piece, while the magnetizing coils, each of which contained about 970 turns of heavy wire, were excited by a current of about 10 amperes. The light from an arc lamp, *L*, passed through a vertical slit a few inches from the lamp and fell upon the mirror, from which it was reflected to a narrow horizontal slit in the slide carrier *S*. The photographic plate was allowed to drop vertically in front of the slit. No lenses were used.

The natural period of the needle was determined by mounting a very small mirror on the end of a heavy tuning-fork which was known to make 512 double vibrations per second, and then placing this fork in such position that its mirror would reflect a spot of light from the lamp to the horizontal slit in the slide carrier beside

[1] Figures 1, 2, 4, 5, and 6 are drawn about one-half natural size.

the spot of light from the mirror on the needle. The needle was deflected by a current from a storage battery, the fork bowed, and the sensitive plate dropped. A trip was so arranged that as the plate passed the slit it broke the battery circuit. By comparing the curve from the fork with that made by the needle as it came to rest, it was found that the needle made seven times as many vibrations as the fork, or about 3580 complete vibrations per second.

Curves were taken from different sources of alternating electromotive force giving different forms, which, when compared with those taken by other methods, showed that the photographed curves could be depended upon, and were probably much more accurate than the others.

In the study of extra currents, some preliminary experiments were made in which curves were taken showing the dying out of current in a closed circuit containing resistance and self-induction when the impressed electromotive force was suddenly removed by short-circuiting the leads from the source. The curves in Figs. 7 and 8 (see Plate) are portions of two thus taken, one for direct current and the other for alternating, the resistance being 5.1 ohms, and the self-induction 0.032 henries. When superimposed they coincide quite exactly. The small sinusoidal curve in Fig. 7 is that of the tuning-fork before mentioned.

Others, not shown, were taken for increase of current when the electromotive force was suddenly impressed. Their form was the same as Fig. 7 would appear if turned over, except that the straight line ended abruptly where the curve began at the instant the electromotive force was impressed.

When taking these curves, the arc lamp was always operated so as to give as bright a spot of light as it was capable of producing. The varying brightness of the curves is probably due to the hissing of the arc.

The final results of the investigation obtained with the galvanometer described were quite satisfactory. These, however, will not be discussed here, since this article relates more especially to the galvanometer itself, using as illustrations a few of the various kinds of curves that have been photographed with it.

Fig. 7.

Fig. 8.

Fig. 9.

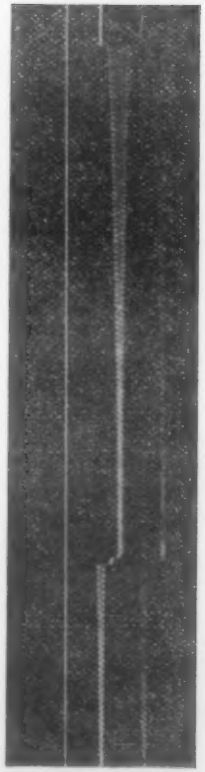


Fig. 10.

Fig. 11.

The galvanometer may be modified to adapt it to the particular use to be made of it. For the study of synchronous motors, new apparatus was made, with modifications, some of which were made necessary by the new conditions, and others tried with a view to improvement and to extending the range of usefulness of the galvanometer. The same electro-magnet was used, but it was necessary to use other coils having only about 1050 turns instead of 1940. Slide *S*, Fig. 3, was made a little longer in order to give the plate a higher speed as it passed the slit. The latter was two feet below the top. The average speed of the plate while passing the slit was about ten feet per second. In place of the arc lamp enclosed in a box, a light wooden box about $8\frac{1}{2} \times 10 \times 12$ inches, lined with asbestos paper, was used to enclose the light and support hand-fed carbons at an angle of about 30° or 35° from vertical, so as to have the maximum intensity directed horizontally toward the narrow slit in the side toward the galvanometer. When fed by hand, curves could be obtained without the variations of brightness seen in Figs. 7 and 8. A space was left between the wood and the asbestos lining, so that air could circulate.

The mode of supporting the needle was somewhat modified, as shown in Figs. 4, 5, 6; Fig. 4 being a plan, 5 an elevation, front view, and 6 a vertical section. For convenience in making, and to get the coil *c* nearer the needle laterally, the coil was supported on two $\frac{1}{4}$ inch rods *rr'*, Figs. 5 and 6, set in holes $\frac{3}{4}$ inch between centers in a piece of wood *d'*. Portions of the rods were cut away as shown. On one a shoulder was left, to which one end of the fiber *f* was fastened, the other end being attached to a spring *m*, which should give as much tension as the fiber will safely bear.

For supporting the coil in position between the pole pieces, *N S*, a standard, *e*, was fastened to a base that was clamped to the yoke piece of the magnet by a screw. The block *d'* is held against the ends of two screws through the standard by tightening the screws in the bent end of the metal strips *k*, which draw upon the rod *h*, through the block. These form a sort of universal joint, convenient for adjusting the needle so as to bring the spot of light from the mirror on the slit of the slide; also, one needle and coil could be readily removed and replaced by another in the experiments made

for the purpose of comparing the behavior of needles of different dimensions and suspensions. The block *d'* served also to hold the binding screws for the small coil.

Several needles were made and tried. Dimensions and data for five of them are given in Table I. The mirrors for the last four were obtained by selecting from a large number of microscope cover-glasses one that had a plane surface, as shown by the interference bands that appeared when the glass was placed on a standard plane surface. The glass was .08 mm. thick.

TABLE I.

MEASUREMENTS OF GALVANOMETERS. DIMENSIONS IN MILLIMETERS.

Numbers.		I.	II.	III.	IV.	V.
Fiber	Material	Quartz.	Quartz.	Quartz.	Quartz.	Silk.
	Length	14.		12.	12.	12.5
	Diameter	0.14		0.015	0.057	0.035(?)
Needle	Length	2.4	Nearly like IV.	1.80	2.50	2.81
	Width	1.25		0.56	1.23	2.56
	Thickness	0.33		0.065	0.07	0.07
Mirror	Length	2.00		0.80	1.72	1.44
	Width	1.25		0.45	0.65	0.49
	Thickness	0.30		0.10-	0.10	0.10
Approximate vibration frequency with about 10 amperes in field coils		3580	3950	3450	3900	2850

After silvering by Draper's method,[1] and varnishing, the thickness was between .09 and .1 mm. It was then cut to the desired size with a small marking diamond.

In mounting the iron and mirror on the fiber, a metal plate was supported a few inches above the table, upon which was fastened one end of the fiber; the other was allowed to hang over the edge, with a lump of wax attached to keep it straight and in place. The iron was placed on the plate under the fiber. Two minute particles

[1] Smithsonian Contributions to Knowledge, Vol. 14, Article IV., p. 3, 1865.

of dry shellac were then placed on the fiber, or close against each side, above the iron, and heat was applied by a bunsen burner beneath the plate until the shellac melted and spread as much as desired. It was then allowed to cool before putting the mirror in position upon the shellac. When the mirror was carefully adjusted, heat was again applied beneath. The mirror was lightly pressed down upon the melted shellac with a plane surface so as not to spring it, and left to cool slowly. The fiber was cut off to the required length and mounted on the supports with melted shellac. The small pieces of iron and mirror may be conveniently handled with two fine pointed glass rods by slightly wetting the end of one and touching it to the corner of the piece to be lifted; then use the dry one to remove the piece from the other, and to place it.

Figures 7–11 inclusive are full size copies of portions of a few curves photographed, the whole plate being $2\frac{1}{2} \times 8$ inches. Figure 9 shows the natural vibration frequency of needle II. for comparison with Fig. 10, which is a current curve for an eight-pole alternator in the Dynamo Laboratory taken with the same needle and 7.8 amperes in the small coil (2) of 34 turns of number 18 copper wire. The plate passed the slit with the same mean speed of about 120 inches per second for both. The frequency of the alternating current was about 98, and that of the needle about 3950 per second. The straight line in Fig. 9 shows the width of spot of light that may be obtained, and that the needle remains perfectly steady while the plate falls if not deflected by current; also that the plate falls without being disturbed by friction or jar. At the end of the short, straight line the circuit from a storage battery was closed by a trip operated by the falling plate holder. When the vibrations due to inertia had nearly died out, the circuit was broken by another trip. In Fig. 10, the parts of a second curve are due to the plate being dropped twice for the same current a few minutes apart. The speed of the dynamo had changed a little.

For part of our work a much slower motion of the plate was desired. This was accomplished by an arrangement on the principle of the Atwood machine. A string hooked to the top of the plate holder passed over a pulley mounted on the top of the slide,

and supported on the other side a weight which nearly counterbalanced the plate holder. A stretched rubber band started the holder and weight, which maintained quite nearly the same speed after the band ceased to act. Figure 11 shows a curve thus taken for electromotive force between two terminals of a three-phase generator on open circuit; the frequency being about 22 cycles per second. Needle IV. was used with the 300 turn coil (3) of

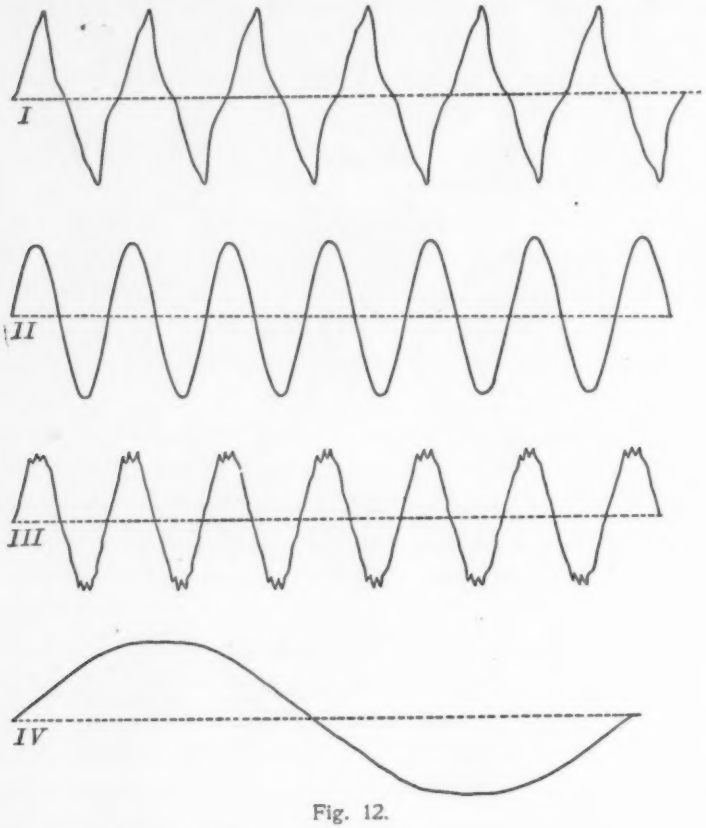


Fig. 12.

number 28 copper wire. A tracing of a current curve for the same dynamo with the plate dropping at full speed is shown a little more than one-half size by IV., Fig. 12. The other three curves are similarly reduced.

In Fig. 12, I. is a curve for current through a synchronous motor run by one of the large transformers in the laboratory.

Curve II. is for current from a small 50-volt, 10-ampere Westinghouse alternator having a smooth cored armature, it being the same one used in the experiments on extra currents.

Curve III. is for current from another alternator of the same type, except that the armature coils are laid in grooves cut in the core. The small variations, plainly noticeable by the shading of the line in the plate, cannot be represented by the tracing. The irregularities of the curve, indicated not only by change of direction

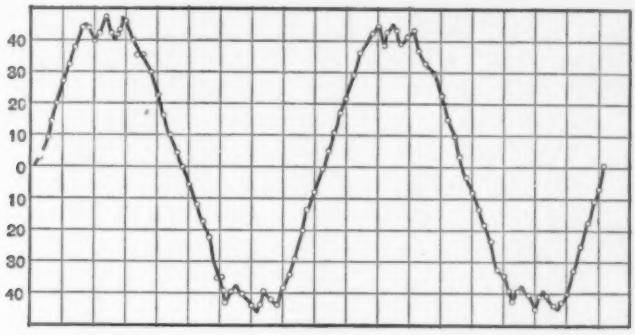


Fig. 13.

of line, but by the shading, recur regularly in each cycle, being of the same number, shape, and position in the cycle; and were evidently not due to the natural period of the needle. Figure 13 is a curve for the same dynamo taken by the instantaneous contact and telephone method.

A photograph was taken the same way as Fig. 9 to try the effect on the frequency of the needle when the field was weakened; and also to show the promptness with which the needle obeyed the deflecting force. With about 12 amperes in the field coils the first throw of the spot of light was 0.55 inch; while with 1.7 amperes in field coils it was 1.3 inches. Under these same conditions the frequency was reduced from about 3900 to 2700. When a square was applied to the angle between the first throw and the axis for zero current, its variation from a right angle could not be detected near the axis. The spot of light in the first throw and return travelled about 2.6 inches, while the plate travelled $\frac{1}{23}$ inch in about $\frac{1}{2700}$ second. For taking curves the slide was usually between 12 and 18 inches from the needle.

To determine whether the deflection of the needle is proportional to current in the small coil, for constant field excitation, a calibration curve for needle III. was taken, with the result shown by line *D*, Fig. 14, which is a straight line; showing that the galvanometer may be depended upon to give ordinates proportional to current when properly adjusted. The needle should be adjusted relative to the pole-pieces until the spot of light remains at rest when the field current is suddenly made or broken.

When using different needles it was noticed that with one there was very little deflection when there was no current in the field

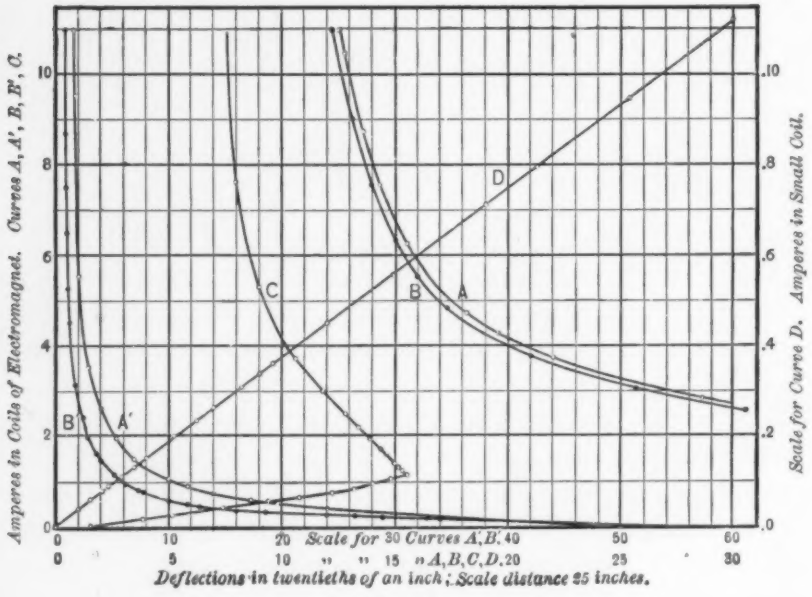


Fig. 14.

coils, while for another the deflection, under the same conditions, was very large. This led to taking the curves shown in Fig. 14, and further explained in Table II.

The three curves, *A*, *B*, *C*, were taken for the same current and with the same conditions, except the use of different needles. Before beginning the readings, the residual magnetism was that left after breaking the circuit when 9 or 10 amperes were flowing in the coils. The cores and pole-pieces being of wrought iron,

it was quite small. The magnetizing current was then gradually increased, giving the curves as plotted.

In order to get the complete curves like *A* and *B*, so as to show the magnitude of the deflections for zero field-current relative to

TABLE II.

EXPLANATION OF CURVES IN FIGURE 14.

Curve.	Needle used.	Constant current in small coil of 300 turns.
A	III.	0.5 ampere
A'	III.	0.06 "
B	V.	0.5 "
B'	V.	0.034 "
C	IV.	0.5 "
D	III.	Constant current in magnet coils=9.4 amperes.

those for higher magnetization, the curves A', B' were taken for smaller currents in the small coil.

From the curves A, B, A', B', it seems that for constant current the action and sensitiveness of the smallest needle with the fine quartz fiber is about the same as that of the comparatively large needle with the silk fiber; and that, for both, the maximum deflection corresponds to a magnetization equal to or less than the residual value. Curve *C* indicates that the coarse fiber is so stiff that a much smaller deflection in a stronger field is the maximum. Other things not yet experimented upon may have a part in determining the form of curve *C*. A study of the behavior of different needles under different conditions with comparatively low magnetic fields is in progress. If sufficient steadiness and freedom from vibration can be obtained by the use of a permanent magnet, or a small electromagnet, it would be much more convenient for a portable apparatus and would be very much more sensitive for experiments in which only small currents can be used.

For one part of the synchronous motor experiment a galvanometer giving a large deflection for small currents of comparatively slow variations was desired. For this purpose a U-shaped perma-

nent magnet of about $\frac{7}{8}$ square inch cross-section was mounted in such a way that any one of the five needles could be readily adjusted; needle V., however, was made especially for use with this magnet. There were four small coils on the permanent magnet which could be used to increase the field if found necessary. The pole pieces were loose and rested on a block to which the needle block was fastened. Either the pointed ends or the square ends could be placed against the coil around the needle, or drawn away from it if a weaker field were desired.

In the spring of 1895 Messrs. A. Barnes and H. Zimmerman undertook[1] an experimental determination of the variations of temperature at the inner surface of the walls of an engine cylinder during a double stroke of the piston; also the variations at different depths below the surface. The use of a quick-acting galvanometer was suggested to them by one of the writers. After a preliminary trial with the large electro-magnet and needle II., they decided to adopt the method; so the one described above with needle V. and coil 3 was constructed and used for the purpose. The general method was to have the changes of temperature produce a corresponding change in the resistance of a fine wire, and thus to vary the amount of current flowing through the galvanometer. The latter was placed in shunt with the resistance. The plate was moved up and down in front of the slit by a cord connected to a reducing motion operated by the movement of the piston, so that the spot of light from the galvanometer traced out a closed curve for the complete cycle, quite like an indicator diagram.

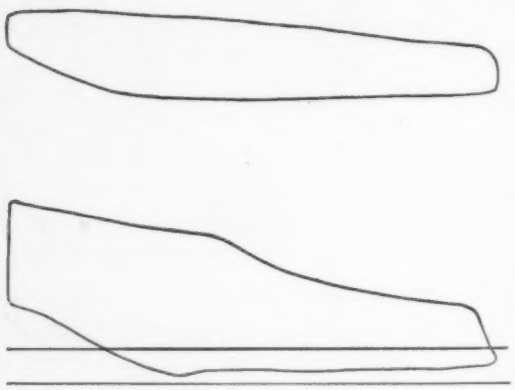


Fig. 15.

[1] Thesis in Cornell University Library; Abstract in Sibley Journal of Engineering, June, 1895.

A tracing from one of them, and a slightly reduced copy of the indicator card taken at the same time, are shown in Fig. 15. The abscissæ for both are the same, representing parts of the stroke. The ordinates of the upper loop represent temperatures, while those of the lower are pressures. The gauge pressure was 40 pounds, and the speed 84 r.p.m.

The vertical width of the upper loop shows the range of variation of deflection, while the total deflection due to the current of 0.4 or 0.5 ampere flowing was about 5 or 6 inches; the plate was perhaps 18 inches from the galvanometer. Each 0.1 inch of change of deflection was found to represent a change of temperature of 12.5° C. The needle trembled slightly, making the finely toothed line about $\frac{1}{40}$ inch wide, the line itself being perhaps $\frac{1}{100}$ inch wide. The width of loop decreased as the speed increased, the results being quite uniform, and apparently in accord with theory. The galvanometer seemed to be quite reliable without any field excitation. For part of the work the pole pieces were turned with the square ends toward the coil, and were also drawn apart to a width of $\frac{3}{4}$ inch instead of $\frac{15}{32}$ inch, as before. The deflections were increased, but the trembling was also increased, so that there was not much gained.

The vibration due to inertia does not take place to any great extent, even with a rather weak field, if the current is continually varying without very large or sudden changes in the rate of variation; for then the condition for simple harmonic motion — that the displacement be proportional to the acceleration — does not exist, if the period of the needle differs from that of the current or its harmonics. It was found that the needles with quartz fibers were more reliable than those having silk fibers, especially for high frequencies and strong fields. For these conditions it is better to have the fiber as large as can be used without too much decrease of deflection or increase of size of the small coil.

It was found that with needles II. and IV. the spot of light was improved by placing a small lens of 10 or 12 inches focal length immediately in front of the small coil, so that the light passed through it both to and from the mirror. The lens was supported on an arm from the standard *e*, shown in Fig. 3. With the other

needles no lens was used. If the mirror is plane, a real inverted image of the arc is obtained at all distances, the principle being the same as in the case of a pin-hole camera. This fact may be used to test the mirrors after they are cut and before and after mounting, to determine whether they have been sprung in mounting and whether they are suitable for use.

In the construction of the apparatus described prominent considerations were that it be inexpensive and easily made from materials, and with means, most readily obtained. Other considerations would doubtless have led to a quite different construction. It was deemed best, however, to investigate the working of the cheap and simple apparatus for special uses, before undertaking a more expensive and elaborate design for more general use.

The original plan for the synchronous motor experiment was to photograph on the same plate simultaneous records of current and lag variations by having the spots of light from two galvanometer needles thrown upon the slit in the slide. The work along this line is not yet completed; however, there does not seem to be anything to prevent the use of two or more galvanometers, or needles for obtaining simultaneous curves for current, electromotive force, etc., on the same plate.

The results thus far obtained seem to warrant a more careful study, and such modifications in design as will adapt the instrument to the many uses to which it may be applied.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
May, 1895.

MINOR CONTRIBUTIONS.

Experiments with a New Polarizing Photo-Chronograph as Applied to the Measurement of the Velocity of Projectiles.[1]

By Albert Cushing Crehore and George Owen Squier.

In May, 1894, a paper[2] was read at the general meeting of the American Institute of Electrical Engineers in Philadelphia describing a novel method of measuring the variations in an electric current. The point of novelty consisted in the fact that the indicator or the vibrator, which actually describes the curve representing the current, possesses no mass, and is therefore not subject to the laws of inertia. It is the application of a modification of this instrument to the measurement of the velocity of projectiles which forms the subject of this paper.

The instrument, as used for measuring current, made use of a dark band in the spectrum to indicate by its movement along the spectrum the varying values of the current. This band is obtained as follows: Instead of taking sunlight direct and analyzing it into the colors of the spectrum, it is first passed through a polarizer and then through a quartz plate to rotate the planes of polarization of the component colors in different amounts depending upon the color, and finally through an analyzer, whose office it is to extinguish only that color which is rotated to the same extent as its plane. The resulting light emerging from the analyzer is colored, due to the absence of the color which the analyzer abstracted from the white light. Upon analyzing the emergent light into the pure colors of the spectrum, the color which the analyzer abstracted is absent, and in its place is a dark band in an otherwise continuous spectrum. The motion of this band is effected by placing between the polarizer and analyzer a transparent substance surrounded by a coil of insulated wire; that substance to have the property of rotating the plane of polarization when in a magnetic field. This field is produced by the current to be measured flowing in the coil of

[1] For a detailed account of these experiments, see the Journal of the United States Artillery, July, 1895, published at Fort Monroe, Va.

[2] "A Reliable Method of Recording Variable Current Curves," by A. C. Crehore; Transactions of the American Institute of Electrical Engineers, Vol. XI., No. 10. October, 1894; also The Physical Review, Vol. II., No. 2. 1894.

wire, and its strength is consequently proportional to this current, since no magnetic material, such as iron, is present.

It may not be evident at a glance what the relation is between an instrument which will accurately measure a variable electric current, and the measurement of the velocity of projectiles. In regard to this it may be said that any good instrument for measuring a variable current possesses the essentials of a chronograph, and a chronograph is the essential instrument for measuring the velocity of projectiles. A good current measurer must give sufficient data to construct a curve, the horizontal axis representing time, and the vertical axis current. The time interval between any two phenomena can therefore be measured by such an instrument if the phenomena referred to are capable of either interrupting the current or changing its strength in any way. The instrument referred to is therefore adapted to the measurement of the velocity of projectiles; for screens may be placed at any desired intervals along the trajectory, and the current mechanically interrupted. The instrument measures the time between these screens by indicating when the current is changed; the distance is measured on the field. The average velocity of the projectile is this distance divided by the time interval.

In the present application to the measurement of the velocity of projectiles, since it is only necessary to indicate the presence or absence of a current, the instrument may be simplified by omitting not only the quartz plate, but also the arrangement for resolving the light into a spectrum. The operation of the instrument is now as follows: White light is passed through the polarizer, and then through the transparent substance possessing the property of rotating the plane of polarization when in a magnetic field, and then through the analyzer. A lens is used to intensify the light emerging from the analyzer by bringing it to a focus on the photographic plate. The analyzer is then "crossed" and set for total extinction, so that normally no light is admitted to the plate. When a current is sent through the coil, all the planes are rotated, the blue more than the red. The effect is that light immediately emerges from the analyzer, and is recorded on the plate. The light persists as long as the current flows, but is cut off completely upon its interruption, — much more suddenly than it could be interrupted by any mechanical shutter. In fact, the office of the whole combination of polarizer, analyzer, and medium for rotating the planes of polarization is to play the part of a shutter.

The experiments with this instrument were made at the United States Artillery School, Fort Monroe, Va., between the dates of Dec. 27, 1894, and Jan. 12, 1895. The instruments used were homemade and quite hastily assembled. Many things adopted for the sake of expediency were a disadvantage in experimenting, and these things, if instruments were now manu-

factured for the purpose, might be avoided. The camera was made at Dartmouth College and the instrument shipped to Fort Monroe during the latter part of December, where it was installed after the 27th inst. Besides this, many things remained to be done after this time, such as running wires to the temporary proving grounds, arranging suitable supports for the novel kind of ballistic screens used, etc., etc.

The camera is shown in the views Figs. 1 and 2. [See Plate.] It consists of a rectangular box *A*, 10 × 10 × 2½ inches inside measurement. The cover *B* is removed, showing a small auxiliary dark chamber *C*, which contains the electromagnetic device *D* with armature *E* attached to the spring *F* for releasing the camera slide *G*. This slide is shown withdrawn from the grooves in which it normally slides by passing it through the opening at the top after removing the cover *H*. The narrow horizontal slit through which the light is admitted to the plate is shown at *I*. The slit is made of sheet brass, the upper jaw being stationary and parallel to a radius of the plate. The lower jaw *J* is a sector of sheet brass which slides between two guides so as to make the slit always a sector of the plate, the object being to secure a uniform exposure for every point of the plate. When the camera slide is in position, the nail at *K* rests on the top of the brass spring *F*, and the upper edge *L* of the lower part of the slide covers the slit. When the current passes through the magnet by the binding posts *M*, the slide is released. The slit is only exposed while the opening in the camera slide is passing by. The upper part of the slide *G* is capable of adjustment along the rods of the slide, and the time of exposure of the slit thus under control. When the cover *B* is in position, the space containing the release mechanism is a complete dark chamber in itself. A cap *N* in this cover *B* is removed just before the camera is to be used. The wires at *O* are for the purpose of producing on the plate reference circles by casting their shadows. The entire back of the camera is removable, and its outside face is shown at *P*. Through the back a horizontal shaft *Q* passes which revolves in the bearing *R*.

The inside of the camera is seen in Fig. 2. The slide is shown with the upper part *G* removed. The plate is mounted on the shaft at *S*.

The object of the camera slide is to prevent the exposure of the plate from extending over more than one revolution and thus spoiling the record by a second exposure.

The polarizer used was a Nicol prism the dimensions of which are 104 by 39 mm. on the side, while the diagonals of the ends measure 60 by 49 mm. The analyzer was similar to the polarizer. Between polarizer and analyzer was placed a tube 45 cm. long and 3 cm. internal diameter, made of glass and fitted with plane glass ends. This tube was filled with liquid carbon bisulphide, which has the property of rotating the plane of polarization when in

a magnetic field, and only then. The tube was wound with No. 18 single cotton magnet wire from end to end in four sections. Each section had 725 turns, which made in all 2900 turns when the four coils were connected in series.

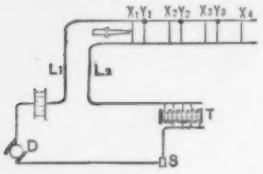


Fig. 3.

The diagram of the circuit which contained this tube is seen in Fig. 3. Current is supplied at a constant electromotive force of 110 volts by the dynamo *D*. The amount of current is regulated by a bank of resistance lamps. The tube is at *T* and a switch at *S*. Line wires L_1L_2 lead to the proving ground, where the screens $X_1X_2X_3X_4$ are erected in the path of the projectile. At $Y_1Y_2Y_3$ are placed devices for establishing the current immediately after it is broken by the preceding screen. The device to re-establish the current is a simple arrangement represented in Fig. 4. The springs *CC* are separated by a small insulating plug *D*, which is attached to a wire running across the path of the projectile. This is mechanically pulled out by the projectile, which therefore establishes the current by connecting the binding posts *BB'* together.

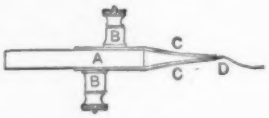


Fig. 4.

The camera shaft was coupled directly to a small electric motor operated by four cells of storage battery. A fly wheel was placed on this shaft, which served a double purpose. It was a gear wheel having fifty-six teeth. By holding the edge of a card against it, a tone is given out whose pitch depends upon the speed. It was thus that the speed was adjusted before each shot. The speed was not obtained in this way, but it is desirable to have a speed such that the plate will revolve nearly once around while the slide is passing by the slit. The note thus obtained was compared with a tuning fork in the other hand of the observer just before each shot, and the negatives show that this method gave pretty uniform results.

The record of the time is recorded on the plate itself at the time the shot is fired. A tuning fork mounted to run electrically is placed so that the shadow of one prong is brought to a distinct outline on the plate. This shadow gives a sinusoidal wave on the negative. On the average about 35 waves of the fork used can be counted on the negative. The convenience of this method proved to be very great, not only because the record of time is on the plate itself, but because it was not necessary to time the revolutions before and after firing.

The tuning fork used made 512 complete vibrations per second, and was in unison with another similar fork before it was mounted electrically. After it was mounted, it was observed to beat with the other fork, so that

its vibrations afterward were calculated to be 509.46. This makes the average time of exposure of the negatives about .066 of a second. A speed which will make the plate revolve once during the exposure is thus about 12.25 revolutions per second, or about 735 per minute.

The complete arrangement of the instruments showing the electrical circuits used with the different pieces of apparatus is shown in diagram, Fig. 5, in which D is the dynamo, T the transmitter tube, S a switch to complete the transmitter circuit a moment before firing to prevent heating the coils, L_1L_2 the line wires leading to the proving ground, $X_1X_2X_3$, etc., the screens, $Y_1Y_2Y_3$, etc., the devices for restoring the current successively

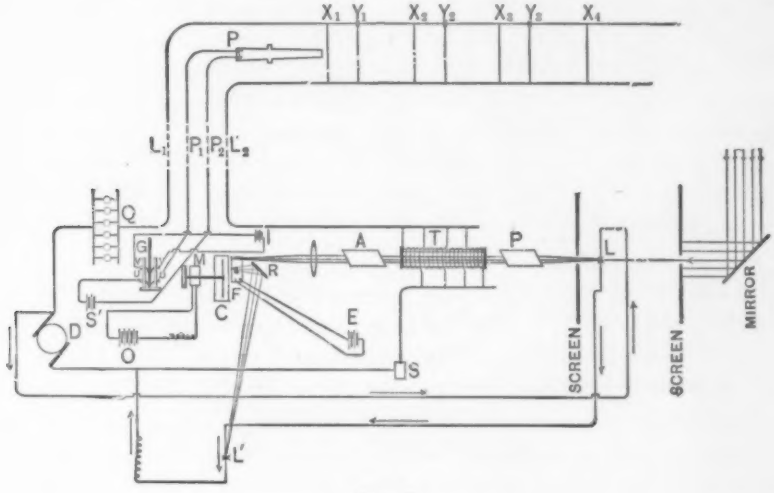


Fig. 5.

between the screens, and L and L' two arc lamps in series, which for convenience were lighted by the same dynamo D. F is the electrical tuning fork controlled by the cells E, and M is the motor for running the camera; and at O are the four storage cells for energizing the same. G is the gravity switch for exposing the camera and firing the gun, and C is the camera whose slide is operated by the cells I through the gravity switch at VV. The firing circuit contains the electric primer P at the gun, the line wires P_1P_2, the dry cells S', and the gravity switch terminals UU.

The shortness of the time of exposure was the cause of the greatest experimental difficulty; for to obtain a record of the projectile, it must be made during the particular .066 of a second when the camera is exposed. Of course a camera made in the future for this purpose can be so arranged as to avoid this difficulty. There is a certain unknown interval of time between the closing of the primer circuit, which fires the gun, and the

arrival of the projectile at the muzzle, which is known as the "firing interval." There is also an unknown interval between the closing of the camera circuit and the exposure of the plate. The relation between these intervals must be determined before the camera can be used to obtain a record. It turned out that the camera circuit should be closed a short time before the primer circuit, to bring the muzzle record at the beginning of the exposure. A gravity switch was constructed with the object of keeping this position when once found constant. It consisted of a brass rod standing vertically between two uprights, and a cylindrical brass weight four inches long with a hole through its axis to permit it to fall down the rod. Near the bottom of the rod are two pairs of contacts, connected to the two uprights having projecting springs so that the falling weight will first "make" the camera circuit and then the primer circuit. The time interval between is adjustable by dropping the weight from different heights. When the proper height was found, this switch gave satisfaction.

The gun used was a 3.2-inch B. L. field rifle, No. 56, model of 1892, and the service charge of $3\frac{3}{4}$ lb. of I. K. H. powder was uniformly employed. The projectiles were common shell, so selected that each weighed 13 lb. 6 oz.

Length of bore of gun	25.2 calibers.
Travel of projectile in bore	21.81 "
Powder chamber capacity	108.9 cu. in.
Density of loading	0.95315.

For the gun two siege platforms were laid in prolongation and leveled, giving a suitable direction of fire out to sea. The firing was conducted without applying the wheel breaks, and the recoil was approximately constant at 48 ft. total, or 28 ft. on the platform and 20 ft. on the ground.

The arrangement of screens for this work possesses some interesting features. A skid $12'' \times 12'' \times 15'$ long was placed in approximate prolongation of the axis of the bore of the gun elevated at 3 degrees, being supported by two solid upright posts. Beyond this, shifting planks were placed end to end, spiked together, and supported by scantlings set into the ground at intervals. The first shots with the gun showed that the blast was not to be feared so much as anticipated. The screens were made of two strong, upright pieces, spiked to the sides of the skid. Along the straight edge of these pieces on the side towards the gun, wire nails were driven in at close intervals, and wire wound around the nails and stretched back and forth, three times being usually sufficient. These screens were placed at various intervals, always beginning at the muzzle of the gun itself. Care was taken to place the muzzle screen at a distance in front of the muzzle equal to the length of the projectile, so that the first screen is not broken until the projectile is out of the bore of the gun.

Figure 6 is a reproduction of the first negative obtained when the circuits were arranged as in Fig. 5, the first screen being at the muzzle and the second 40.13 ft. from the first. The record of the muzzle screen appears at X_1 on the negative, the make at Y_1, and the second break at X_2. The angle between the breaks can be read with considerable accuracy. It was measured with a large spectrometer, the circle of which is graduated directly to ten minutes, and with the verniers the reading is to ten seconds. The plate is mounted horizontally on the turning table, and centered by means of the reference circles at *A* and *B* put on the plate for the purpose by the wires in the camera at *O*, Fig. 1. On this plate an average of nine readings of the angle makes it 108° 5′.815 ± .′444 or 108°.0969 ± .°0074. The probable error shows that this angle is measured with an accuracy of .0068% of the whole, or of one part in 14,630. Of course the percentage of error depends upon the whole angle measured, but 40 ft. is a smaller distance than it is customary to use with other chronographs. The tuning-fork record shows that an angle of 294°.54 subtends just 34 waves of tuning-fork, seen at the *CD*, Fig. 6. The time of one wave being $\frac{1}{509.46}$ of a second, the angular velocity of the plate is 4413°.42 per second. The velocity of the projectile is

$$v = s/t = \omega s/\theta,$$

where s is the distance between screens, θ the angle between breaks measured on the plate, and ω the angular velocity. Upon substituting the values found, the velocity is found to be 1638.5 ft. per second.

It was next attempted to obtain observations at several points along the trajectory, at regular intervals, at first fifteen-foot, then ten-foot, and lastly five-foot intervals were tried, to a distance of 45 ft. in each case. Records with all of the intervals above mentioned were successfully obtained, and in Fig. 7 is reproduced a negative taken, showing five-foot intervals beginning at the muzzle and extending to a distance of 45 ft.

It is interesting to note that when the interval between the screens is so small as 5 ft., it is necessary to place the device to establish the current before the previous screen, as the projectile moves four or five feet while the jaws of the springs are coming together. Measurements on this plate give the following angles: —

s	θ	s	θ
5	13° 49′.29	30	82° 28′.25
10	27° 19′.29	35	— —
15	40° 53′.79	40	110° 10′.46
20	54° 36′.00	45	123° 50′.88
25	68° 24′.83		

The relative velocities calculated from these angles considering the muzzle screen as the first one in each instance, are

s	v	s	v
2.5	1599.4	15.0	1608.3
5.0	1610.8	17.5	—
7.5	1621.7	20.0	1605.3
10.0	1619.5	22.5	1606.5
12.5	1615.7		

If the five-foot screen is taken as the first, the same angles give the following velocities : —

s	v	s	v
2.5	1599.4	17.5	1610.5
7.5	1637.5	20.0	—
10.0	1633.0	22.5	1606.0
12.5	1626.8	25.0	1607.3
15.0	1619.7		

These tables are exhibited graphically in Fig. 8, which shows that the velocity increases outside of the bore of the gun up to a certain maximum point at a distance of six or seven feet from the muzzle. This same effect is exhibited on all the negatives obtained with five-foot intervals, and they all compare well together. The increase is so great, about $2\frac{1}{2}\%$ of the whole, that one cannot attribute the increase to errors.

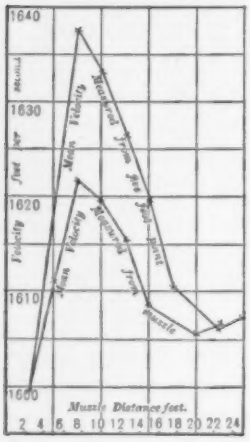


Fig. 8.

The reason why the points do not lie on a smooth curve is of course because of errors, but it is more probable that the error arises from the fact that the projectile does not strike consecutive screens exactly alike, and thus the intervals are not precisely five feet, than that they are due to the measurement of the angles on the negatives. So it appears that there is now greater accuracy in measuring the time interval than in finding the exact distance.

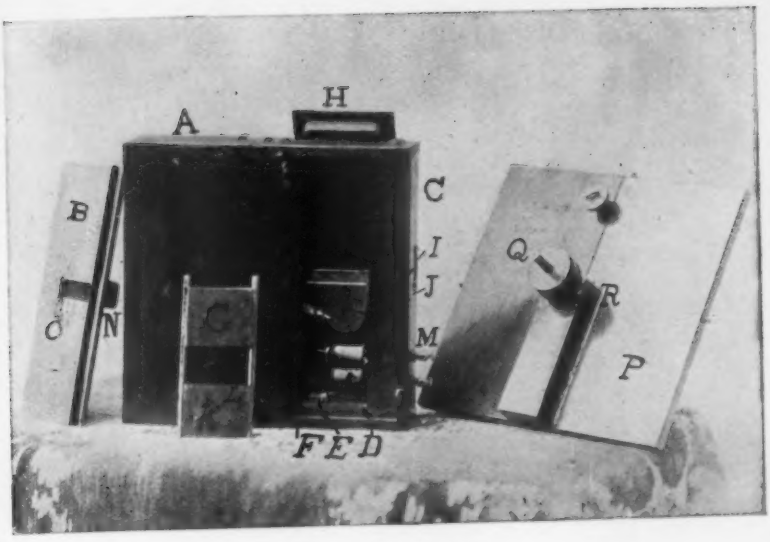


Fig. 1. The Camera.

Fig. 6. Chronograph Record.

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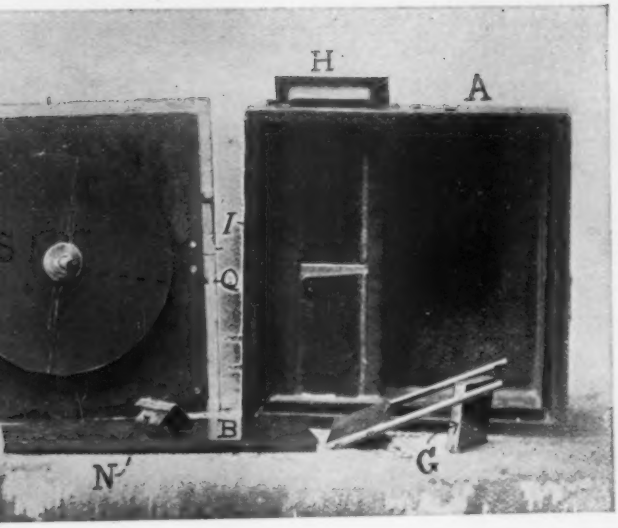


Fig. 2. The Camera.

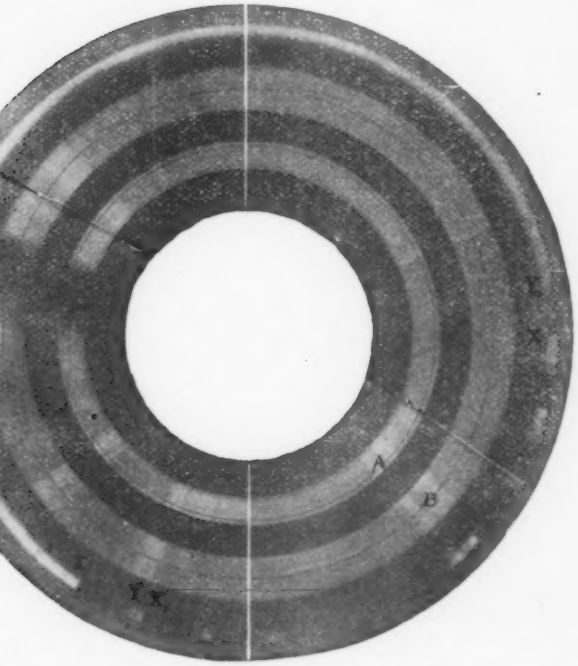


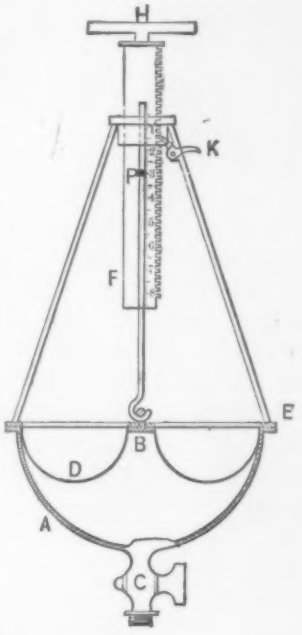
Fig. 7. Chronograph Record.

G PHOTO-CHRONOGRAPH.

Experimental Demonstration of a Law of Fluid Pressure.

By W. J. Humphreys.

IT is well known that the pressure at any point of a perfect fluid is equal in all directions, whether the fluid be at rest or in motion, and that this equality of pressure is also true of viscous fluids, provided they are at rest. Of course it can be rigidly demonstrated mathematically that this is a consequence of the inability of fluids to sustain a tangential stress, but such a demonstration is beyond the grasp of the average student when he first begins the study of physics; consequently an experimental demonstration becomes useful. The piece of apparatus here described for demonstrating this principle is easily manipulated and is well adapted to classroom work.



It would be easy to criticise several experiments suggested by elementary text-books for the demonstration of this law of fluid pressure, but I will simply say that one of the best I have seen is that of exhausting a pair of Magdeburg hemispheres, and then showing that they are held together in whatever direction they be turned. This shows that there is pressure in every direction, but does not show how much in any direction.

The piece of apparatus which I constructed for this purpose consists of a single Magdeburg hemisphere *A* (see figure), to which is connected a stopcock *C* for exhausting air, and a spring dynamometer *F* for measuring the pressure exerted. As shown in the illustration, a thin but strong rubber diaphragm *D* is made fast, by means of a metallic rim *E*, to the top of the hemisphere. To the center of this diaphragm is fastened a metallic button *B* to which is hooked the rod that communicates with the spring dynamometer. The dynamometer can be placed at any desired distance from the diaphragm and held in that position by means of the rack and catch *K*. The apparatus was so constructed that it was easy to remove an old and put on a new diaphragm, though occasion for this seldom occurred.

The experiment is performed as follows: Place the dynamometer as near the diaphragm as it will go, exhaust the air from the hemisphere by means

of the stopcock to any degree desired (this can be done sufficiently well by the mouth), and then by means of the handle *H* raise the dynamometer to a suitable position and fasten it with the rack and catch. The index *P* will now show a certain pressure which will remain constant, no matter in what direction the rubber diaphragm be turned, up, down, or sidewise at any angle; thus experimentally demonstrating that within the limits of delicacy of the apparatus the pressure of the atmosphere — and presumably of other fluids — is at any place equal in all directions. Evidently apparatus of the same general plan could be used to demonstrate this law in the case of liquids.

I used this device in the physical lecture room of the Miller Manual Labor School, Crozet, Va., for several classes, and it always gave entire satisfaction.

JOHNS HOPKINS UNIVERSITY, April, 1895.

NEW BOOKS.

Die Principien der Mechanik in neuem Zusammenhange dargestellt. Von Heinrich Hertz. 8vo, pp. xxvii, 307. Leipzig, Johann Ambrosius Barth, 1894.

The work before us possesses a melancholy interest, not only as being the crowning labor of the short life of its gifted author, but as being preceded by a most sympathetic and appreciative preface by his great master Helmholtz, who was so soon to follow his favorite pupil. To have been the favorite pupil of Helmholtz was in itself no small honor, but to have justified the expectations of the master with such completeness as was the case with Hertz, was indeed extraordinary. Helmholtz, great and clear-sighted as he was in all the domains of knowledge that he touched, can hardly be said to have founded a school, for although he had many clever and appreciative pupils, few seem to have been so imbued and penetrated with the ideas of Helmholtz as to have followed in the creative work characteristic of him. The single exception is Hertz. The whole work of Hertz shows the influence of Helmholtz. The philosophical consideration of fundamental principles, the frequent introduction of metaphysical considerations, the logical and systematic deductions from the premises, and the exceedingly general nature of the principles enunciated, all bear the stamp of the master. This is true in a large degree of the present work, although the manner and matter are highly original.

In a preface of fifteen pages Helmholtz gives an account of the life of Hertz, and a review of the development of the science to which he made the greatest contributions. What Helmholtz thought of Hertz may be gathered from the following quotation: "Favored with the rarest gifts of mind and character, in his too short life he gathered a full harvest of almost unhoped-for fruits, for which during the preceding century many of the most talented of his fellow-scientists had striven in vain. In ancient classical times it would have been said that he had fallen a sacrifice to the envy of the gods. Here Nature and fate seemed to have favored in a quite uncommon manner the development of a mind which united in itself all the faculties necessary to the solution of the most difficult problems of Science. . . . I myself felt this grief deeply, for among all the pupils that I have had I might consider Hertz the one who had penetrated most

deeply into my own circle of scientific thoughts, and upon whom I might place the surest hopes for their further development and enrichment."

When Hertz finished school at the age of eighteen, he chose the profession of an engineer, being, with characteristic modesty, somewhat doubtful of his success should he take up theoretical science. This doubt lasted two years, after which he began the study of physics under Helmholtz at Berlin. The latter immediately recognized his talent, and at the end of the second semester, having to propose the subject for a prize in physics, chose a subject in electrodynamics, in the hope that Hertz would interest himself in the subject and attack it with success. This subject was no less than a research intended to throw light on the critical question of action at a distance, as opposed to action from particle to particle. The various theories of Weber, Ampère, Neumann, Riemann, Grossman, and Clausius are characterized by Helmholtz as a "brilliant bouquet of hypotheses, very little 'übersichtlich' in their consequences, so that the domain of electrodynamics had at that time become a pathless desert." Weber had undertaken to remove certain difficulties in his theory by the hypothesis that electricity had a certain amount of inertia. To find out whether this was true was the problem attacked by Hertz. By means of experiments on extra-currents in double coils he showed conclusively that not more than one-twentieth of the extra-current could be due to inertia of electricity. But Hertz was not satisfied with this result. Recognizing that the induction in straight wires could be calculated with much greater accuracy than in coils with many turns, he experimented with wires forming two rectangles, and thus reduced the effect that might possibly be attributed to inertia to less than one two-hundred-and-fiftieth of the whole. Further experiments were still more convincing as to the untenability of Weber's hypotheses. It is easy to see how these experiments were the beginning of Hertz's study of electrical theory. Helmholtz, the first convert and apostle in Germany of the Faraday-Maxwell theory opposing action at a distance, considering the experimental demonstration of the existence of a magnetic effect due to the changes of dielectric polarization in an insulator (Maxwell's displacement currents), to be a crucial point in the decision between the rival theories, made it the subject of one of the great prize contests of the Berlin Academy of Sciences. The result was his now famous research on High Frequency Electrical Oscillations, and the experimental proof of the correctness of Maxwell's theory, making it highly probable that luminous vibrations are of an electromagnetic character.

After the preface by Helmholtz comes the author's preface, in which he states the object of physics to be the explanation of all natural phenomena by means of the simple laws of Mechanics. What these laws are is, however, not a matter of common agreement. It is the aim of the author to

embrace all the laws of mechanics, including Newton's fundamental laws of motion, in some principle that shall be so simple as to be intrinsically probable, the truth of which shall be justified by its agreement with experience. Helmholtz had proceeded in such a manner in his work on the Principle of Least Action, starting with the assumption of Newton's laws and Hamilton's principle. The same ideas appear in the work of J. J. Thomson on the applications of Dynamics to physical phenomena. Hertz, however, proceeds in a different manner, the reason for which he explains in a long philosophical introduction. In this he discusses with great logical acumen the necessary and sufficient properties that must be possessed by any "image" that shall represent to us the true essence of the phenomena of motion, and of such images he considers in detail three, in historical order. In the first system the conceptions supposed given *a priori* are the ideas of space, time, mass, and force. Force is introduced as the cause of motion, existing before and independently of the motion. Kinematics connects the ideas of time and space, and Galileo's conception of inertia gives a relation between space, time, and mass. In Newton's laws appear all four fundamental conceptions, and the principle of d'Alembert gives the general statement of all that is predicated of them. The rest is simply deduced by logical processes. While the system here treated is perfectly logical, and is therefore a satisfactory "image," the conception of force has given rise to a number of difficulties, chiefly of a metaphysical nature, and has in recent times been the subject of vigorous attacks.

The second image treated is the far more modern one in which the notion of energy appears as a fundamental concept instead of that of force. Although this system has been frequently advocated, there exists no text-book of mechanics which follows it out completely, introducing the idea of energy before that of force. In this system the two conceptions of space and time have a simple mathematical character, while the other two, mass and energy, are introduced as given indestructible physical entities. In order to connect all four, there is needed besides the principle of conservation of energy a further principle, which may be taken as that of Hamilton. This would give as the sole result of experience needed the principle that every system of natural masses moves as if it had the task set of reaching given positions in a given time in such a manner that the mean difference between kinetic and potential energy throughout the motion should be as small as possible. This second image avoids the difficulty of the first of speaking of things of whose nature we know little or nothing, and which have also little influence on the result. For in speaking of forces we must ultimately come to speak of forces between individual molecules and atoms, of which our notions are by no means directly derived from experience. The following quotation may possibly impress others as agreeably as it did the reviewer.

"It affected almost painfully so rigorously thinking an investigator as Gustav Kirchhoff, to see atoms and their vibrations thrust into the midst of a theoretical deduction without dire necessity." Such a statement is refreshing in these days when every student, nay, almost every dynamo-tender, talks so glibly of molecules and of the ether. "The arbitrarily assumed properties of the atoms may be without influence on the final result, the latter may still be correct. Nevertheless, if the details of the deduction itself are in a large degree presumably false, the deduction is illusory. The older conception of physics leaves us here hardly any choice or mode of escaping the difficulty. On the contrary, the theory of energy and our second image of mechanics possess the advantage that there appear in the statement of the problem only things directly appreciable by experience, parameters, or arbitrary co-ordinates of the bodies considered; that the discussions proceed with the help of these quantities in finite and complete form, and that the final result can be immediately translated back into terms comprehensible by experience."

Hertz does not, however, adopt this second, or energy-image, partly because, like any other, the principle of Hamilton is not of perfect universality, partly because the principle does not possess the simplicity of enunciation requisite for a fundamental principle, as well as for more or less metaphysical reasons. The third image, to which the present work is devoted, differs from the others in that it contains but three fundamental conceptions instead of four. These are space, time, and mass. The fact that these three were sufficient was made prominent by Kirchhoff in his philosophical treatment of mechanics in his celebrated text-book. In order to fill the apparent gap without introducing a new conception, Hertz supposes that in addition to the masses that are perceived by our senses, there act certain "concealed" masses, which are recognizable only by the effect that they produce on ordinary bodies. Such bodies are the ether, Lord Kelvin's and Maxwell's vortical fluid, and the matter treated by Helmholtz in his work on "Cyclic Systems." This hypothesis has the advantage not only of eliminating all mysterious forces from mechanics, but also of preventing their entrance at all. In carrying out this principle, Hertz makes the further supposition that the connections between parts that act upon each other are rigid, thus returning to the idea that preceded that of action at a distance. Unfortunately, as Helmholtz remarks, Hertz does not give examples to explain how he conceived such intermediate members between bodies not in contact, and it will probably require a large expenditure of physical imagination in order to explain even the simplest cases of physical forces. Physicists of such extraordinary intuitional powers as Maxwell and Kelvin would probably have no difficulty in so doing. It is comforting to know that Helmholtz himself obtained more satisfaction by the ordinary

methods of considering the facts as represented by the differential equations of physics.

The first half of the body of the book, treating of kinematics, concerns itself merely with definitions and terminology. This is very carefully worked out, and although appearing very abstruse, is found, when the idea of the author is once seized, to be of considerable simplicity. Ideas generally considered as applicable to the motion of a point are generalized in such a manner as to apply to systems composed of any number of material points. The length of a displacement of a system is defined as the quadratic mean (square root of mean square) of the displacements of all its mass-points, every particle of mass m being supposed in taking the mean to consist of m separate mass-points. It is easily shown that the distance or displacement between two positions of a system is always less than the sum of the distances of the two from a third, and that consequently it is possible to represent three such distances by the sides of a triangle. Any displacement may thus be considered as the resultant of two others, and may be represented by a single vector. This vector does not, of course, give us the position of each point of the system, but it has a definite relation to the configuration of the system. We may, by means of the triangle, define the difference in direction between two displacements. In particular we may define the amount and direction of infinitesimal displacements, and proceeding from one configuration to another we may consider the path of a system. The path is defined as straight if its direction is constant, which will be easily seen to involve the condition that the paths of every particle of the system are straight in the ordinary sense. If the path is not straight, we may define its curvature as the rate of change of direction per unit length of path traversed. An element of a path is straighter than another if its curvature is less. A straightest element of path is one that is straighter than all other possible elements having the same position and the same direction. A path all of whose elements are straightest elements is called a straightest path.

These definitions being premised, we may state the fundamental principle enunciated by Hertz, the consequences of which are elucidated in the second half of the book treating of dynamics.

Every free system persists in its state of rest or of uniform motion in a straightest path.

Systema omne liberum persevarare in statu suo quiescendi vel movendi uniformiter in directissimam.

This law, which is merely a generalization of Newton's first law by the changes necessary in substituting the word "system" for "material point," contains in itself all three laws. From these are immediately deduced the integral of energy, and Hamilton's principle. As all the definitions have

been free from reference to any particular system of co-ordinates, the equations of motion are obtained in generalized co-ordinates in the form given by Lagrange. The motion of systems under geometrical constraint is then treated, and finally the motion of systems under the action of forces. Force does not here appear as a fundamental notion, but simply as a sort of analytic expression for the amount of deviation of a system from the straightest path, accompanied by an equal and opposite deviation of another system. This conception is after all not different from that of Kirchhoff, but the enunciation of it is here extremely precise and lucid.

The aim and general method of the work before us have now been described. There follows a treatment of the important case of cyclic systems, following Helmholtz, concealed cyclic motions, illustrating the fundamental supposition, and finally treatment of discontinuous motions. Whether the method of deducing the fundamental equations employed by our author is preferable to the usual methods may be considered doubtful — it seems to be merely a question whether we shall take a little more or less for granted. The present reviewer must confess that he feels rather content to be satisfied with the representation of the physical facts by the equations than with an attempt to reconcile them with a simple principle, and to confine his flight far below the upper air of metaphysics; and he is glad to be supported by the opinion of Helmholtz already quoted. However this may appear to others, the work of Hertz shows that it is not without reason that in German universities natural science finds its place in the *philosophical* faculty, and we can but admire the clear insight and the logical exposition of a system, which evince the treatment of a most difficult subject by a master mind.

ARTHUR G. WEBSTER.

Elementary Lessons in Electricity and Magnetism (New Edition). By SILVANUS P. THOMPSON, F.R.S. 8vo, pp. xv+628. Macmillan & Co., 1895.

An elementary text-book in science which has lived fourteen years, and which has had a sale of a number of thousand copies each year, must be a remarkable book, deserving the attention of all teachers. The book before us is no less than this, and in a completely revised edition will be welcomed by all who are interested in technical education. This book has no doubt been used as the first step in the electrical instruction of a majority of American technical graduates who are now employed in the electrical industries, and numberless men holding responsible offices will remember with interest their study of "Thompson's Lessons" during one of the terms of their sophomore year at college.

The new edition of this book bids fair to be as useful as the old. It has been brought fairly up to date, and on the whole has been made to conform very well to our present methods of scientific thought. The book has more and possibly better competitors for the attention of students and teachers than formerly, but none are so clear and satisfactory for the reader. There are one or two text-books that probably make a better foundation for the sophomore lectures and laboratory practice in electricity and magnetism which are presented to the students of our technical schools (Chapter XVI. of the revised Daniell's Physics, for instance); but, while these give a better preparation for the advanced work of the technical student, they are, as a general rule, too heavy for the less strongly prepared general student or the undirected reader. The reviewer is not a teacher of physics, but he feels safe in saying that Thompson's Lessons has no real rival as a text-book in the general course in college, or for a place in the hands of the general reader who wishes to obtain a knowledge of electricity and magnetism.

For elementary course instruction, the additions which have been incorporated in the new edition relating to dynamo calculations, working of alternators, theory of transformers, etc., are of questionable utility, but for the general reader these articles must be of considerable interest, and may possibly be useful. It is always possible for a reviewer to find defects in a book, especially if he is inclined to resort to special pleading, but the defects in the book before us are amply compensated by virtues. It is unfortunate that the author's proof reading was not more carefully done, especially with a view to bringing the phraseology of the old portions of the book into harmony with the more modern forms of the new. Many slips in spelling might also have been corrected; but the most prominent fault of the book is to be found in its definitions, which may sometimes be misleading to an inexperienced student. This is particularly unfortunate in a beginner's book, as a young student is readily led into error or thrown into perplexity by statements which are perfectly plain to a more experienced reader. The use of the erroneous term "virtual" instead of the formally adopted term "effective," when referring to alternating volts and amperes, must be strongly criticised. Alternating current phenomena are such a stumbling-block to the average student throughout his studies, that it is especially desirable to avoid any artificial addition, however small, to the difficulties which may be caused by a double use of terms, and it is to be hoped that the word "virtual" will be replaced in the plates by "effective" for the benefit of future reprints.

The mechanical execution of the book is carried out in the excellent style for which the publishers are justly noted.

DUGALD C. JACKSON.

Steam and the Marine Steam Engine. By JOHN YEO. 8vo, pp. xiv+196. Macmillan & Co., 1894.

This work is professedly intended for naval officers and for students of engineering in the earlier part of their training.

With this end in view, the size of the book has been kept within small limits, and the evident purpose has been to give, in plain, readable style, the salient features of modern marine engineering practice.

Judged from this standpoint, the work seems to have been conscientiously done, and the selections from the broad field of engineering practice have generally been made with care and judgment. The treatment of propulsion and resistance is, however, exceptionally brief, the latter being dismissed with less than one page, presumably on account of its less vital connection with the main subject in hand.

The illustrations are numerous and excellent, and the make-up of the book is in the usual good style of the publishers.

On the whole, there seems to be little to criticise adversely, and much to commend, when viewed from the standpoint of the author's avowed purpose. Viewed more broadly, however, the extent of the field of usefulness for such works may be fairly called in question. No book of 200 octavo pages can satisfactorily present even the leading features of so broad a subject as that here dealt with; and, while such works may serve as an introduction to more extended study, or for those who only wish a general outline of the subject, they must be laid aside early in the *student's* course to give place to sources of more extended and exact information.

W. F. DURAND.